Minna Liechti.
141 Spring Garden Road
Halifax.

Bertha Liechti.
Ditto.

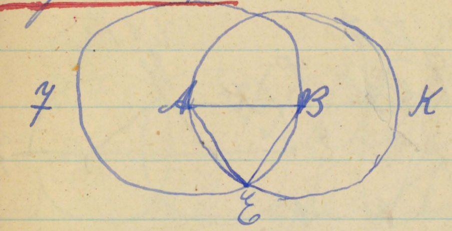
Minna Liechti.

Bertha Liechti.

Oct. 27th 1887.

Exercises in Geometry.

Page 12 ex 1.

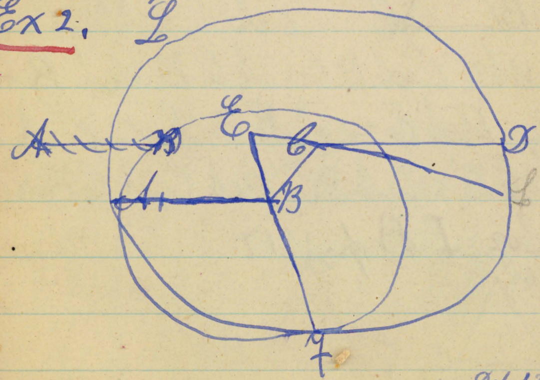


∵ A = centre ⊙ BEF ∴ AE = AB

∵ B = " ⊙ AEK ∴ BE = AB

∴ AE, BE, AB are all = ∴ ABE is an equilat. △.

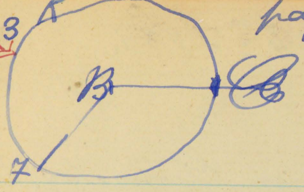
Ex 2.



E.D. = EF CD = BF radii ⊙ ∵ Def 13. AB = BF

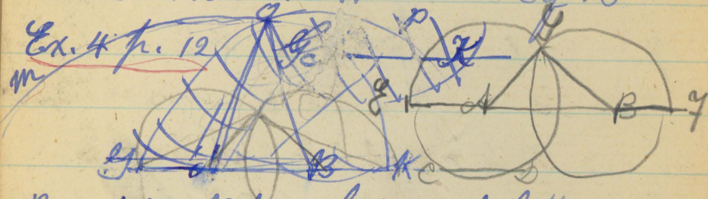
∴ AB = CD.

Ex. 3 page 12



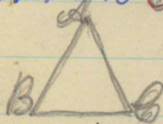
centre B dis. BC ⊙ from B draw BF to Oce ∴ BC = BF.

Ex. 4 p. 12 Def 13



Prod AB & BA each to equal GK centre G desc. ⊙ GHMO centre K desc. ⊙ OPG. join OA & OB

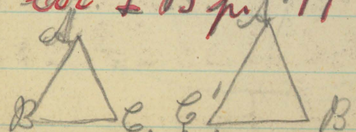
Cor. I A. p. 16



(I A)

∵ AB = AC ∠ ABC = ACB AB = BC ∴ ∠ BAC = ∠ ACB but ∠ ABC also = ACB ∴ ABC, BAC, ACB all =

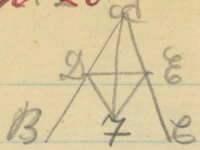
Cor I B p. 17



∵ ∠ ABC = ∠ A'C'B' ∠ ACB = A'B'C' BC = C'B' ∴ Δs = (I B) hence AB = A'C'

AC = A'C' ∴ AB = AC

Ex. I p. 20



∵ AD = AE ∴ ∠ ADE = ∠ AED (I A)

∵ DF = EF ∴ ∠ EDF = ∠ DEF (I A)

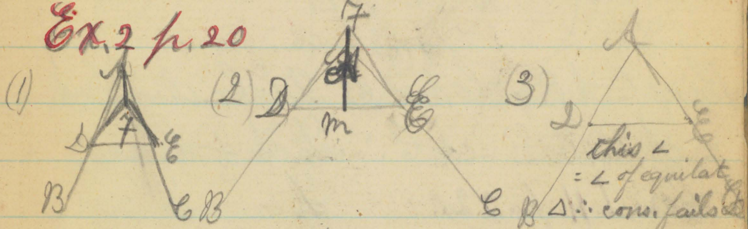
∴ sum ∠s ADE, EDF = sum AED, DEF

in Δs ADF & AEF ∵ AD = AE

DF = EF ∠ ADF = AEF ∴ Δs = (I4)

∴ ∠ BAF = ∠ CAF ∴ ∠ BAC bisected

Ex. 2 p. 20



(1) ∵ AD = AE & AF common DF = EF

∴ ∠ DAF = ∠ EAF (I C) ∴ ∠ BAC bisected

(2) ∵ DF = FE, AF common & DA = AE ∴

∠ DFA = EFA (I C) in Δs DFM & EFM

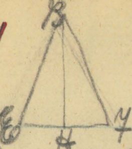
∵ DF = EF, FM common ∠ DFM = ∠ EFM

∴ DM = EM (I4) in Δ DAM, EAM

∵ AD = EA, AM common DM = EM.

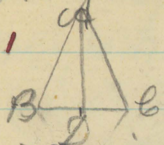
∴ ∠DAM = ∠EAM ∴ ∠BAC bisected.

Ex. 1. p. 21



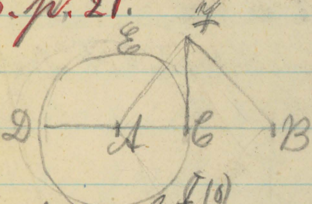
△s = (I 4) ∴ EH = HF

Ex. 2 p. 21



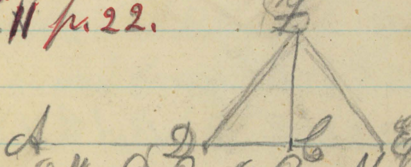
3 sides △s = ∴ ∠BAD = ∠CAD (I 8)

Ex. 3. p. 21.



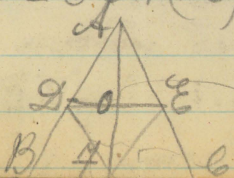
Bisect AB in C desc ⊙ DEC then DA = AC & CB = AC ∴ DA ⅓ line.

Cor. I 11 p. 22.



∴ DF = EF & DC = EC & FC common ∴ ∠DCF = ∠ECF (I 8) ∴ FC ⊥ AE Def 10

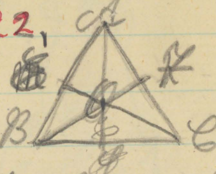
Ex 1 p. 22



In △s ADO & AEO ∵ AD = AE & OD = OE, AO common, ∴ ∠AOD = ∠AOE (I 8) ∴ AO ⊥ ED (Df. 9)

Ex AD = AE, AO common, ∠AOD = ∠AOE ∴ OD = OE (I 4)

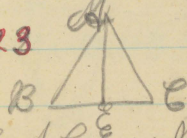
Ex. 2. p. 22.



In △s AEO & BEO = (I 4) (Ax 1)

By (I 4) AO = OC. And AO = BO ∴ all =

Ex. 2. p 23



In △s ABE, ACE ∵ BE = EC, AE common ∠BEA = ∠CEA ∴ AB = AC (I 4) ∴ △ is isosceles

Ex. I p. 23

Ex. 3. p. 23

In △s FAB & EAB, ∠FAB = ∠EBA & FA = EB & AB common ∴ AE = FB (I 4)

In △s CFB & BDC, ∠DBC = ∠FCB & FC = DB, & CB common ∴ FB = CD (I 4)

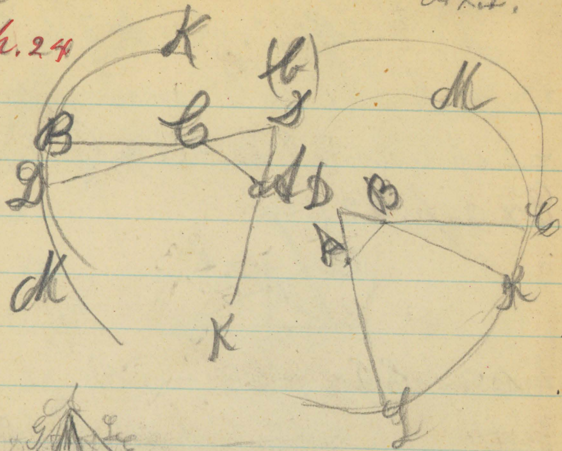
AE = FB & CD = FB. ∴ AE = CD

∴ all = Ax. I.

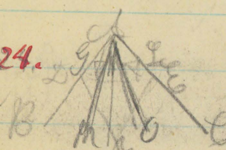
Ex. 1 p. 24

(a)

(b)



Ex. 2. 24.



Prove by IX

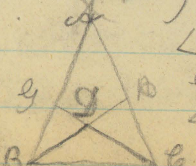
Ex. 4. 24

In Δs FBD & EDC ∵ FB = DC, BD = EC ∠ FBD = ECD ∴ FD = DE I 4

" Δs FBD & FAE ∵ FB = AE, BD = Ad ∠ FBD = FAE ∴ FD = FE (I 4) FD = DE

FD = ~~FE~~ ∴ all = (Ax I)

Ex. 3 p. 24

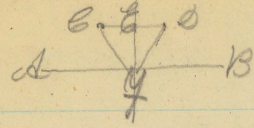


∠ ABC = ∠ ACB each bisected ∴ ∠ DBC = DCB ∴ DB = DC (Con. B

DBC = ...

Ex. 5 p. 24

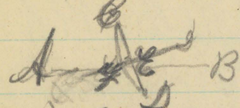
(a)



∵ CE = ED & EF common ∠ CEF = ∠ DEF

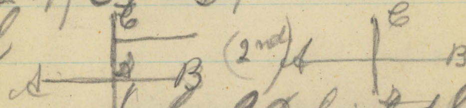
∴ CF = FD (I 4)

(b)



(by I 4) CF = DF

Fail



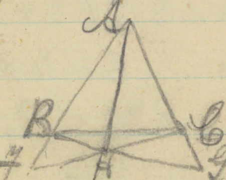
(1st) (2nd)

{when CD bisected by AB

Ex. 6. p. 24.

by (I 4)

Ex. 7. p. 24.



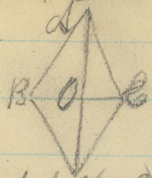
In Δs BAG & CAF ∵ BA = CA

AF = AG ∠ cons. BAG = CAF ∴ BG = CF

Δs BFC BCG by (I 6) ∠s BCF & CBG = that is CBH = BCH ∴ by

~~Cor. I B.~~ BH = CH.
In △s BAH & CAH by (C) ∠BAH
= ∠CAH ∴ ∠BAC is bisected by AH

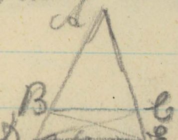
Ex. 8 p. 24.



In △s ABD & CAD by (I C) ∠BAO = CAO
" △s BAO & CAO by (I 4) ∴ BO = CO ∠BOA
= COA ∴ (Def. IX) AD bisects BC
@ rt. ∠s.

Ex 9. An infinite number.

Ex 10.



Make BD = CE join DC & BE
complete △. In △s DBC & BEC ∵
BD = CE, BC com. ∠DBC given = ∠BCE
∴ DC = BE (I 4) In △s BDE & CDE (I 6)
∠BDE = CED that is ∠ADE = AED
∴ AD = AE Cor. B parts BD = CE
∴ rem. AB = AC (Ax 3) ∴ ABC isosceles

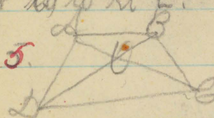
Ex. 11 p. 24.

Join CD. ∵ AC = AD ∴ ∠ACD
= ADC. ∠ACD greater BCD (Ax 9)
∠BDC greater ∠ADC ∴ BD
greater ∠ ACD ∴ BD not = BC

Ex. 12 p. 24.

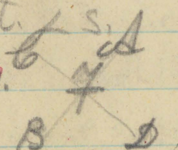
In △s ADC & AEB ∵ AD = AB
AE = AC & EAB com. ∠DAC BAE ∴ DCA
BEA & DCA is a rt ∠ by construction
∴ BEA is a rt ∠.

Ex 1 p. 25



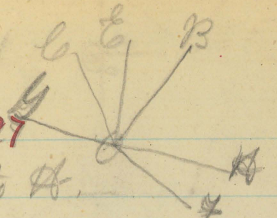
∵ AO meets DB ∴ ∠AOB & BOC (2 rt ∠s) (I 13)
∵ DO meets AC ∴ ∠DOA & COD 2 rt ∠ (I 13)
∴ sum = 4 rt ∠s.

Cor I (I 15) p. 27.



A7 meets CD ∴ ∠A7C & A7D = 2 rt ∠s (13)
and ∠B7C & B7D = A7D and A7C (I 15)
∴ all = 4 rt ∠s

Cor. 2 (I 15) p. 27



Prod GO to H.

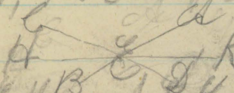
∵ EO meets GH ∴ ∠s on one side GH

= 2 rt ∠s (I 13) ∵ FO meets GH ∴ FOH

& FOG = 2 rt ∠s. ∴ sum ∠s GOC, COE,

EOB, BOF, FOG = 4 rt. ∠s.

Ex I p. 27.



K ∠s CEB & AED

= (I 15) ∴ CEH = ∠AEK

∠CEH, HEB, CEA = ∠ CEH, CEA, AEK but

first 3 = 2 rt ∠s ∴ HEK is st. line.

Ex 2 p. 27.

Make FE rt. ∠s to AB at E & make

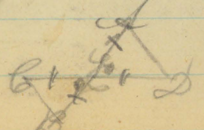
GE at rt. ∠s to CD at E then ∠ GEA

= AED. ∠s FEA & GED each rt. ∠

take away common ∠ GEA ∴ rem. FEG

= AED.

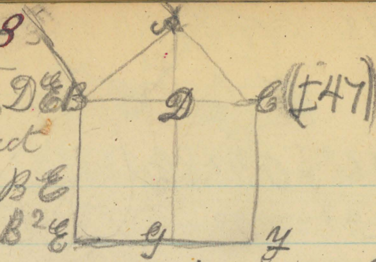
Ex 3 p. 27.



By (I 4) AD = CB

∴ Δs = equal.

Ex. 1 P. 93



AB^2 = rect. DEB

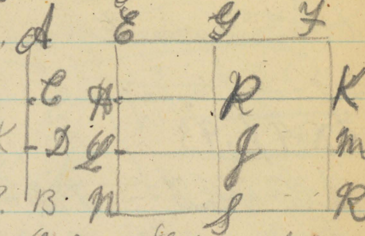
rect. DE = rect

CB. BD ∵ BE

= BC ∴ AB^2 E

= rect. BC. BD.

Ex 1 p. 78



ER = sum of EP, GK

HJ. PM. JN. JR.

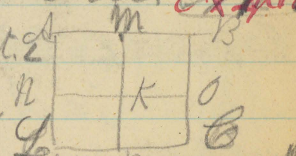
rect. ER = rect. AB. EF ∵ EN = AB

∴ EP = AC. EG ∵ EH = AC

" HJ = CD. EG ∵ HL = CD

∴ LS = DB ∵ YN = DB etc.

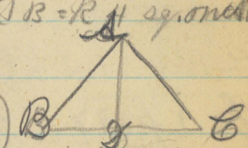
Ex 2 p. 78



AM = AN ∠ MAN rt. ∠

∴ AK = a sq. similarly

others are sqq. ∴ sq on AB = 4 sq. on AM



$AD^2 + BD^2 = AB^2$ (I 47)

$AD^2 + DC^2 = AC^2$ (I 47)

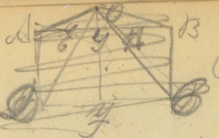
$2AD^2 + BD^2 + DC^2 = AB^2 + AC^2 = BC^2$

$= BD^2 + DC^2 + 2BD.DC$. $2AD^2 = 2BD.DC$

∴ $AD^2 = BD.DC$.

Ex. p. 81

Ex. 3. p. 93.



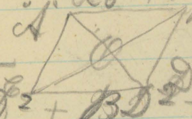
Prove that $OA^2 + OC^2 = OB^2 + OD^2$

$OA^2 + OC^2 = OG^2 + GA^2 + OF^2 + FC^2$

$OB^2 + OD^2 = OG^2 + GB^2 + OF^2 + FD^2$

take away common sqq. & $AG^2 = GB^2$ & $DF^2 = FC^2$ ∴ $AC^2 + OC^2 = OB^2 + OD^2$.

Ex. 4 p. 93

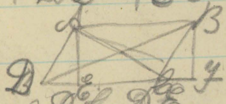


$AD^2 + BC^2 =$

$AC^2 + CD^2 + BD^2 + DB^2 =$

$CB^2 = AC^2 + BD^2 + DC^2 = 2AC^2 + AD^2$

Ex 2 p. 93



$AB^2 =$ (13 II)

$AD^2 + DC^2 - 2DE.DE$

$DB^2 = BC^2 + CD^2 + 2DC.CF$ (12 II)

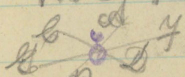
In △s ADE & BCF ∵ ∠BFC = ∠AED

AD = BC, & AE = BF ∴ DE = CF (I.26)

∴ 2DC.DE = 2DC.CF ∴ $AC^2 + BD^2$

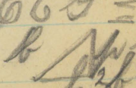
$= AD^2 + DC^2 + BC^2 + AB^2$ = sqq. on 4 sides

Ex. 2 p. 27



∠FEC = FEC ∠ & EC + CEG =

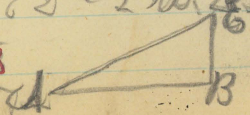
FEC + FED = 2 rt. ∠s

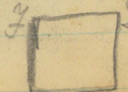


$h^2 = b^2 + p^2$

$h^2 - b^2 = p^2$

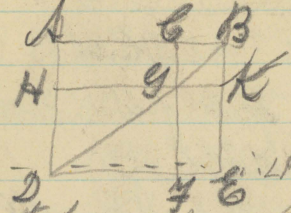
Ex 5. p. 93





ex.

ex. 1. p. 85



In △HGD

∵ HG = HD ∴ ∠HGD = HDG (I.5)

& ∠DHG rt.∠

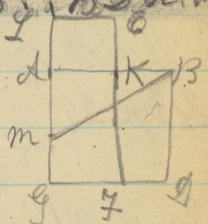
∴ ∠HGD = $\frac{1}{2}$ rt.∠ (I.32) & ∠GCB = rt.∠

In △CBG ∵ CG = CB

& ∠CGB = CBG ∵ each $\frac{1}{2}$ rt.∠

∴ ∠s HGD & CGB + HGC = 2 rt.∠s ∴ BD st. line

ex 1 p. 89



rect AB.BK = AK^2 (II 11)

$AB^2 + BK^2 = 2AB.BK + AK^2$ (II 7)

$= 2AK^2 + AK^2$

$= 3AK^2$

Ex p 90

$AC^2 = AD^2 + DC^2 + 2$ rect. $ED.DC$

$BD^2 = BC^2 + DC^2 + 2\,CF.DC$

$AC^2 + BD^2 = AD^2 + BC^2 + 2\,DC.DC$

$+ 2\,CF.DC = AD^2 + BC^2 + 2\,DC.DC$

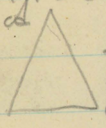
$+ 2\,ED.DC + 2\,CF.DC$

$= AD^2 + BC^2 + 2\,DC(DC + ED + CF)$

$= AD^2 + BC^2 + 2\,DC.CF$

$= AD^2 + BC^2 + 2\,DC.AB$ (I 34)

Ex. p. 31

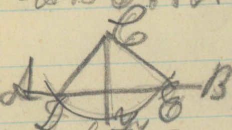


If ABC g. $AB \angle ABC$ g $\angle ACB$

ABC. $AB \angle ABC \angle ACB$

but $\angle ACB = ABC$ ∴ $AB = AC$ (I 18)

Ex 1 p. 29.

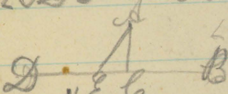


From C draw CF ... CF = to one another (I 16)

if possible make CF = $CD = CE$ ∴ $\angle CDF = CEF$ $\angle CFD$ g CEF

$\angle CFD$ g $\angle CDF$ ∴ CF not $= DC$.

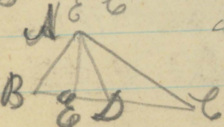
Ex 2. p. 29



$\angle AEB$ g $\angle AEC$ ... AEC less rt. $\angle$

AED, AEC = 2 rt. $\angle$s

Ex. p. 91



∴ AED more rt $\angle$

∴ AED obt. AEB acute

Let ABC be Δ D middle pt. join AD

from A draw $AE \perp BC$ prove AB^2 ...

$= AD^2 + DB^2 - 2\,BD.DE$ (II 13)

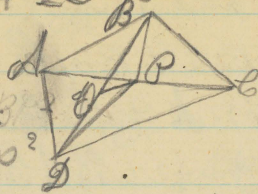
$AC^2 = AD^2 + DC^2 + 2\,ED.DC$ (II 12)

but $BD = DC$ ∴ $2\,ED.DC = 2\,ED.DB$ ∴ cancel

∴ $AB^2 + AC^2 = 2AD^2 + DB^2 + DC^2$

$= 2AD^2 + 2DB^2$

Ex. 6 p. 93.



$AB^2 + BC^2 =$

$2AP^2 + 2BP^2$

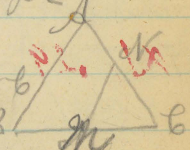
$AD^2 + DC^2 = 2DP^2 + 2AP^2$

∴ $AB^2 + BC^2 + AD^2 + DC^2 = 4AP^2 + 2DP^2 + 2BP^2$

$= 4AP^2 + 4BO^2 + 4OP^2$

$= AC^2 + 4BD^2 + 4OP^2$

ex. 1 p. 41



Diff. AB & $AM = NC = AC$ $AN = B$

∴ MN & BM = diff MC & MN = less NC

∵ 2 sides g. third ∴ diff = less third ex. 2 p. 33

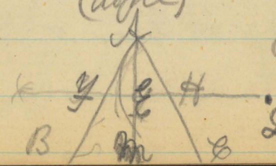
∴ diff. BM, MN = less NC

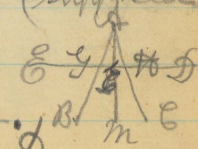
Ex 2 p. 41 $\angle ADB$ g $\angle DAC$ (I 16)

$\angle BAD = DAC$ ∴ ADB g BAD

∴ AB g BD similarly (supposed) (dope)

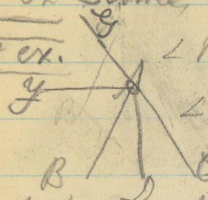
Ex 3 p. 41

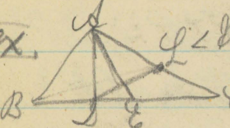


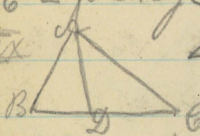


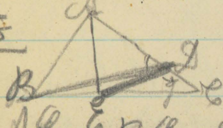
bis. ∠ BAC. Make from D. DF ⊥ to AE
AH = AF in △s AHE, AFG ∵ AH
= AF, AE common ∠ AEH = AEF ∴ EH = EF

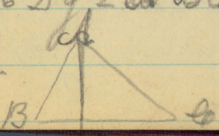
4th ex Same, and ∴ ∠ AHC = AFE.

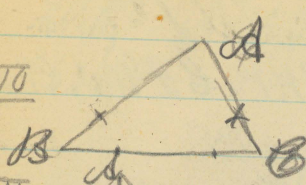
5th ex.  ∠ BAG, BAC = 2 rt.∠ (I 13)
∠ BAF $\frac{1}{2}$ ∠ BAG, ∠ BAD
sim
= $\frac{1}{2}$ ∠ BAC ∴ ∠ FAB,
BAD = a rt. ∠ ∵ @ rt. ∠s to one ans

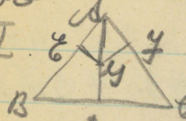
6th ex.  ∠ Make AL = AB ∴ ∠ AE
ABD = ∠ ALD ∠ ADB
= ADE ∠ ADB g ∠ AED ∴ AE g. AD

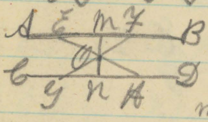
7th ex  ∠s ADB & ADC = 2 rt.∠s
∠ ABD less ADC
∠ ACD less ADB ∴ sum ABD & ACD
= less 2 rt. ∠s.

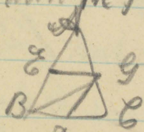
ex 8  In △s ABE & ADE ∵ AB
= AD, AE com. ∠ BAE =
∠ DAE ∴ BE = DE, ∠ ABE = ∠ ADE But
∠ CDE g ∠ ADE (I 16) ∴ g ∠ ABE ∴
∠ ACD g ∠ ABC.

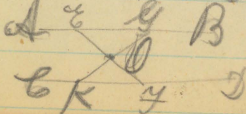
ex 9 

ex. 10  D △ & △

ex. 11  In △s AEG, AFG ∵ AE
com. ∠ AEG = AFG, ∠ EAG =
∠ FAG ∴ EG = FG (I 26)

ex 1 p. 49  O draw MN perp. AB
then
meet CD, MN per. to CD
∠ BMN, MND 2 rt. ∠ BMN 1 rt. ∠ ∴ MND 1 rt.∠
OM = ON △ MEO, NOH, MO = ON, ∠ EMO =
∠ ONH, & ~~EO = OH~~ ∴ ∠ MOE = ∠ NOH ∴ HN
ME. sim. shown MF = GN ∴ EF = GH.

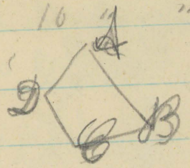
ex 2 p. 49 
~~△ A ABC~~
∠ EBG = GBD, EGB = BDG (I 29) in △s
BEG, BDG ∵ ∠ BEG = ∠ BDG, ∠ EBG = ∠ GBD
& BG com. ∴ EG = GD (I 26)

ex. 3 p. 49 

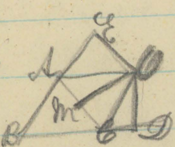
∴ ∠ BDC ...

3 sides have 2 rt. ∠s
4 " " 4 " "
5 " " 6 " "
6 " " 8 " "
7 " " 10 " "

ex.5 p.55



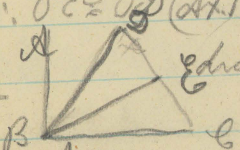
ex.1 p.56



△s AEO, OAM = OE = OM. (I 26)

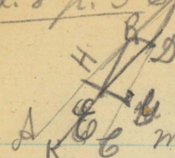
△s OMC, OCD = OM = OD (I 26) OE, = OM & OD = OM ∴ OE = OD (Ax.1.) ∴ OE, OM, OD =

ex.2 p.56



draw on BC equilat. △ BDC ∠ of equi△ = $\frac{2}{3}$ rt. ∠, ABC = 1 rt. ∠ ∴ ABD = $\frac{1}{3}$ rt. ∠ bisect ∠ DBC then ∠s DBE, EBC each = $\frac{1}{3}$ rt. ∠ ∴ ∠ABC trisected.

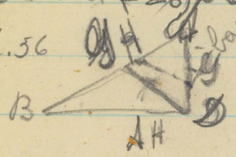
ex. 8 p. 56



KED ∥ AB
DB ∥ AE join BE

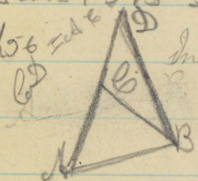
make DG = DL through D BD, BE. HB the DE, BE meets them ∠ HBE = ∠ DEB BE meets HE, BD ∠ HEB = ∠BDE in △s HBE BDE (I 26) = ∴ ∥ BD ∥ EG BD meets ∠BDE = ∠DEG etc. (I 26) EG = BD, HE = BD ∴ AE = EG

ex.9 p.56



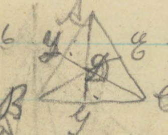
make D blank take H G pt. about G
or bisect AD draw perpen. give one isos. ∠
∠ = ∴ isos.
∠ GDA = GAD ∴ isos. rem ∠BDG = ∠ GBD ∴ △ isos. GB = GD ∴ = AG.

ex. 10 p.56

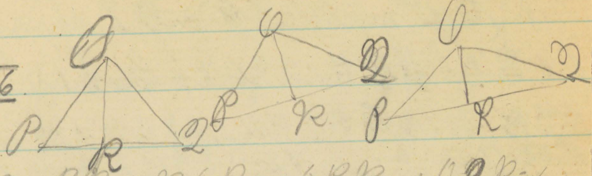


In △s, AC = CD, CB com.

ex. 3 p.56



In △s AD E, AFD, FD = DE (I 26) △s AFD DGC, FD = GC (I 26) ex. 9 p. 56



If OR = PR ∠ROP = ∠OPR. ∠OQR = ∠ROQ, sum ∠s OPR & ROP = sum ∠s ROQ, OQR. right ∠

C

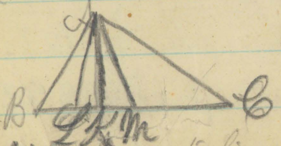
OR g PR, OPR g ∠ROP, g PQ, ∠OQR g ∠ORQ sum OPR, OQR g sum POR QOR sum OPR OQR g ∠POQ ∴ acute ∠

OR less PR ∠POR g ∠OPR, OR less RQ

QOR less OQR, POQ g. OPR, OQR

POQ obtuse ∠

ex. 5



Const. pt. A in st. line AK make ∠KAM = ∠KAL (I.23) ∠KAB = ∠KAC hyp.

∠KAM = ∠KAL (const.) ∴ ∠LAB = ∠MAC (ax. 3). ∠ALC = ∠LAB + B (I.32)

∠ALB = ∠LAC + C (I.32)

∠ALC = ALB (rt. angles)

∴ ∠LAB + B = ∠LAC + C (ax 1)

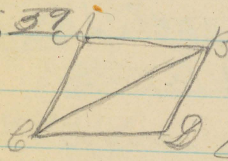
∴ ∠CAB + B = ∠LAK + ∠KAM + MAC + C

∠LAB = ∠MAC

∴ ∠B = ∠LAK + KAM + ∠C

∴ ∠B − ∠C = 2 LAK.

ex. 5. 57



(I.29) ∴ AB ∥ CD

∠ABC = ∠BCD (I.C)

∠ACB = ∠CBD ∴ (I.C)

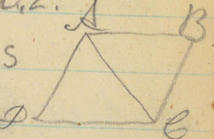
AC ∥ BD (I.27).

ex. 3

BAD, ADC 2 rt ∠s

OAD, ADO = 1 rt. ∠

∴ ∠AOD 1 rt. ∠.



ex. 7 DAB = $\frac{2}{3}$ rt. ∠ (∧ = $\frac{4}{3}$ of 1 rt. ∠) bis. in △s

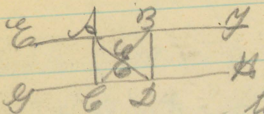
DAC, ABC = (I.4)

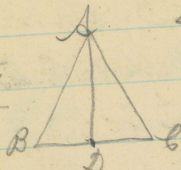
∠s DAB ABC = 2 rt. ∠s ∴ ∠ABC = $\frac{2}{3}$ rt. ∠s

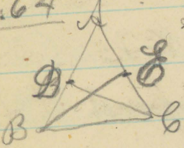
∠BAC, BCA = $\frac{4}{3}$ rt. BAC, BCA each = $\frac{2}{3}$

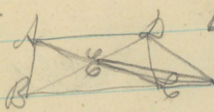
∴ equilat. △ both △s the ∴ 2 equilat.

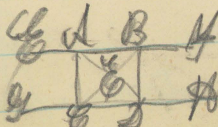
(1) p. 63  $\triangle DPC = \triangle DBC$ (I 37)
$\triangle DBC \frac{1}{2} \square$ ∴ $\triangle DPC$
$= \frac{1}{2} \square$ ∴ $\triangle AP, PBC = \frac{1}{2} \square$.

ex. 2.  $\triangle ACD = \triangle BCD$ (I 37)
take c. $\triangle CED$ $\triangle ACE =$
$\triangle BED$ (ax. 3)

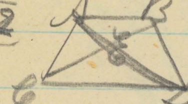
ex 1 p. 64  BC bis. by AD.
$\angle \triangle s =$ (I 38) equal bas.

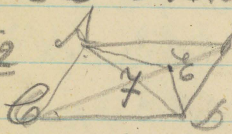
ex 2 p. 64  $DB = AE, DBC = \triangle ABE$
$EC = DB$ $\triangle s =$

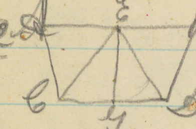
ex. 1 p. 65  $\triangle ABC = \triangle DBE$ (I 37)
$\triangle DBC = EBP$ ∴
$\triangle ABC = \triangle CBP$

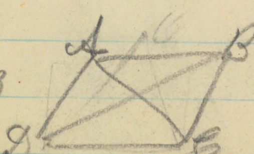
ex 2 p. 65  Add ECD to ACE
BED ∴ =
between same ∥ (I 37)
∴ AB ∥ CD

ex. 1 p. 62  $AD = DB$ ∴ $\triangle ADE = \triangle$
BDE ∴ (I 38) $= BEC$ (ax 1)
$\triangle ADE = \triangle BEC$

ex. 1 p. 72  ∵ $CE = BE$, ∴ $\triangle$
$ACE = \triangle ABE$
∴ etc. $\triangle CED = \triangle BED$. sum $\triangle$s
$ACE, DCE, =$ sum $\triangle$s ABE, DBE
∴ $\triangle ACD = \triangle ABD$ (ax. 2.)

ex. 2. p. 72  ∵ $AF = FD$ ∴ $\triangle ABF$
$= \triangle BFD$ $\triangle AFE$ also
$= \triangle EFD$ take AEF and EFD
∴ sum. $ABE = BED$

ex. 3 p. 72  $\triangle AEC = \triangle EBD$ (I 38)
$\triangle ECG = \triangle EDG$ (I 38)
sum $AEC, ECG =$ sum EBD, EGD,
∴ $ACEG = BEGD$.

ex 4 p. 93 
$AC = AB$ Hyp.
∴ $AC^2 = AB^2$ (ex. 2 p. 73)

$AC^2 + BD^2 = 2AB^2 + 2BC^2$

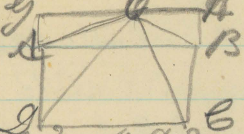
$\therefore AB^2 + BD^2 = 2aB^2 + 2BC^2$

$\therefore BD^2 = AB^2 + 2BC^2$

$\therefore BD^2 = AC^2 + 2BC^2$

$\therefore AC^2 = BD^2 - 2BC^2$

ex 3 p. 93

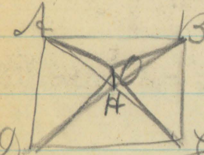


$OA^2 + OC^2 = OD^2 + OB^2$

$OA^2 + OC^2 = OG^2 + GA^2 + OH^2 + HC^2$

$= OG^2 + GD^2 + OH^2 + HB^2$

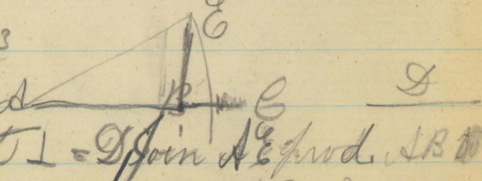
$= OD^2 + OB^2$ (I 47)



$AO^2 + CO^2 =$
$2AH^2 + 2OH^2$
$AH = BH$

$\therefore AO^2 + CO^2 = DO^2 + BO^2$

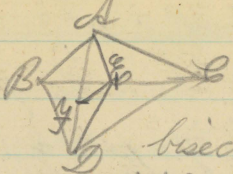
ex. 5 p. 93



At B erect ⊥ = D join AE prod. AB
D, desc. ⊙ cutting AB prod. in C rect. on
sum and diff. AB, AC = sq. D
Diff. of $AB^2\,AC^2$ = diff. $AC^2 + AB^2$

$= BE^2 =$ sq. on D.

ex. 6 p. 93



ABCD quad. bisect diag. E, F

join AE. sum $AB^2 + AC^2 =$
$2AE^2 + 2BE^2$ (ex II 13)

join ED, $BD^2 + DC^2 =$
$2BE^2 + 2DE^2$ (ex. II 13)

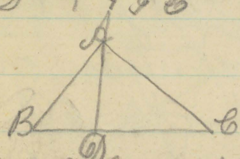
$AB^2, AC^2, BD^2, DC^2 =$
$(2AE^2 + 2DE^2 + 4BE^2)$

$AE^2 + DE^2 = 2FD^2 + 2FE^2 \therefore$

$2AE^2 + 2DE^2 = 4FD^2 + 4FE^2 + 4BE^2$

$= BC^2 + AD^2 + 4FE^2$

ex. 7 p. 93

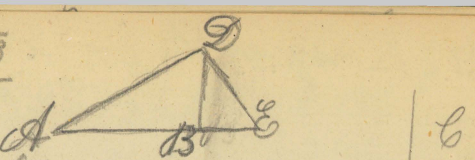


2 rect. $BD.DC = 2AD^2$

$BD^2 + DC^2 + 2BD.DC = 2AD^2 + BD^2 + DC^2$

$\therefore BC^2 = BA^2 + AC^2$

$\therefore \angle BAC =$ rt. $\angle$ (I 48) ex. 8 p. 93



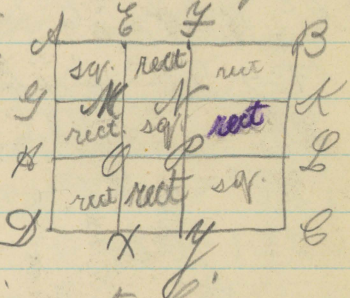
~~make st. ∠ △ prod. AB to E~~

Let AB be line to prod.
At B make perpen = C at D in st. line AD make st. ∠ prod. AB to E join DE.

BD^2 = rect. AB, BE.

BD^2 = sq. on C ∴ line produced = C

ex. 9 p. 93



Let AB given st. line. on AB desc. sq. ABCD. Divide AD into parts similar to AB, through G K draw GMNK & HOPL ∥ to AB, through E F draw EMOX & FNPY ∥ AD.

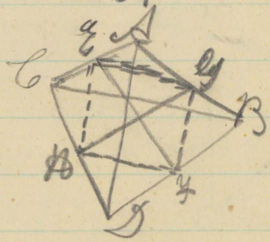
Prove $AB^2 = AE^2 + EF^2 + FB^2$ + 2AE. EF + 2AE. FB. + 2AE. EF

AEMG = AE^2 ∵ GAE rt. ∠ AG made = AE.

GMOH = rect. AE. EF ∵ GM = AE (I 34) GH made = EF.

EF. EM. = rect. AE. EF ∵ AE = EM. NL = rect. EF, FB ∵ NL = FB & NP = EF MP = EF^2 ∵ OMN rt. ∠ MN = EF MO = GH or EF, FK = rect. AE FB ∵ AE = FN HX = rect AE FB ∵ HD = FB HO = AE.

ex 10 p. 93



Let ABDC be quad. AD, CB diag. join middle pts. AB, CD AC, BD, join EH, HF, FG, EG then EF = parallelogram.

∵ GF, EH each ∥ AD and $= \frac{1}{2} AD$

EG, HF each ∥ BC and $= \frac{1}{2} BC$

EF, GH, diag. ▱ EF

$AD^2 + CB^2 = 2EF^2 + 2GH^2$

$EF^2 + GH^2 = EG^2 + EH^2 + FH^2 + FG^2$

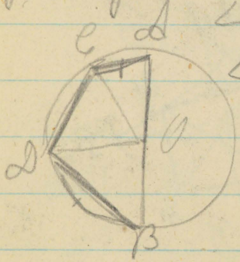
(ex. 2 p. 93) $= 2EG^2 + 2EH^2$ (ex p. 79)

$2EF^2 + 2GH^2 = 4EG^2 + 4EH^2$

$= CB^2 + AD^2$

ex. 9 p. 93 continued

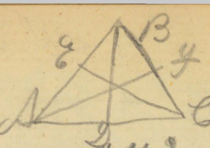
every thing goes wrong today



∠s ACD, ABD = 2 rts

∠ CAB, CDB = 2 rts

ex. 11 p. 93



$AB^2 + AC^2 = 2AF^2 + 2BF^2$

$BC^2 + CA^2 = 2CE^2 + 2EB^2$

$AB^2 + BC^2 = 2BD^2 + 2AD^2$

$2AB^2 + 2AC^2 = 4AF^2 + 4BF^2$

$2BC^2 + 2CA^2 = 4CE^2 + 4EB^2$

$2AB^2 + 2BC^2 = 4BD^2 + 4AD^2$

$4AB^2 + 4BC^2 + 4AC^2 =$

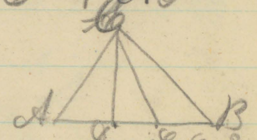
$4BD^2 + 4EC^2 + 4AF^2 + AB^2$

$+ BC^2 + AC^2$

$3AB^2 + 3BC^2 + 3AC^2 = 4BD^2 +$

$4EC^2 + AF^2$

ex 13 p. 93



$AC^2 \cancel{+ CB^2} + CD^2 + CE^2$

$= 2\cancel{AD^2 + 2DE^2 + 2CD^2} + 2CE^2$

$\cancel{AB^2 = 2AD^2} + 2DE^2 + CD^2 + CE^2$

$AC^2 + CE^2 = 2AD^2 + 2DC^2$

$CD^2 + CB^2 = 2DE^2 + 2CE^2$

$AC^2 + CB^2 + \cancel{CD^2} + \cancel{CE^2} =$

$2AD^2 + \cancel{2}DC^2 + 2DE^2 + \cancel{2}CE^2$

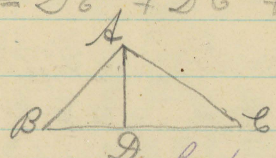
$= 4DE^2 + DC^2 + CE^2$

$= 3DE^2 + DE^2 + DC^2 + CE^2$

$\therefore AB^2 = 3DE^2 + DE^2 + DC^2 + CE^2$

$\frac{2}{3} AB^2 = DE^2 + DC^2 + CE^2$

ex. 14 p. 93



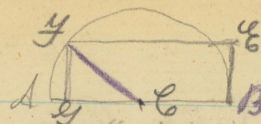
Easy method may be proved

$AD = CD$

$BC^2 = 4CD^2$ (Ex page 79)

$= 4AD^2$ (ax 1)

ex. 15 p. 93



D

~~AB = rect AG, AB = sq. m.~~

Let AB be given st line D less ½ AB.

With centre C and dist. CB desc ⊙ From B draw BE at rt ∠ AB = D Through E draw EF ∥ AB to meet FG at rt. ∠ to AB join FC

$AG.GB + CG^2 = AC^2$ (II 5)

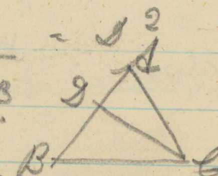
$= FC^2$ (Df 13)

$\therefore AG.GB = FC^2 - CG^2$

$= FG = CB^2$

$= D^2$

ex. 12 p. 93



AB = AC prove $CD^2 = BD^2 + 2AD.DB$

$AC^2 = CD^2 + AD^2$ (I 47)

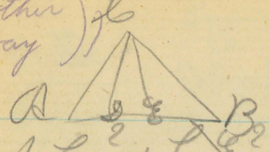
$AB^2 = AD^2 + BD^2 + 2AD.DB$

$\therefore CD^2 + AD^2 = AD^2 + BD^2 + 2AD.DB$

$\therefore CD^2 = BD^2 + 2AD.DB$

((13 p 93))

(other way)



$AC^2 + CB^2 = 2AD^2 + 2DC^2$ II 13

$CD^2 + CB^2 = 2DE^2 + 2CE^2$ II 13

$AD = DE$

$AC^2 + CB^2 = 4DE^2 + CE^2 + DC^2$

$= 3DE^2 + DE^2 + CE^2 + DC^2$

$AC^2 + CB^2 = AB^2$ (I 47)

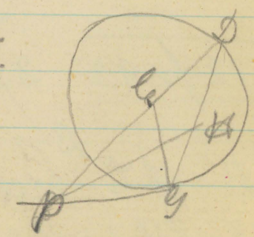
$9DE^2 = AB^2$

$DE^2 = 1/9\, AB^2$

$3DE^2 = 3/9\, AB^2$

$\therefore \frac{2}{3} AB^2 = DE^2 + CE^2.$

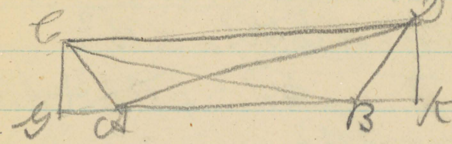
ex. prop XX Bk III



∠ CGP rt. ∠ sum CPG ∠ GPC = 1 rt. ∠

S

ex.



$AD^2 = AB^2 + BD^2 + 2$ rect. AB BK

$CB^2 = AC^2 + AB^2 +$ " " GA. AB

$AD^2 - CB^2 = AB^2 + BD^2 + 2\,GA.AB$

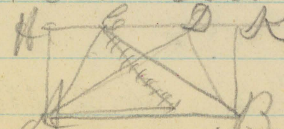
$- AC^2 + AB^2 + 2\,AB.BK$

$= BD^2 - AC^2 + 2AB(GA + BK)$

But $AD^2 - CB^2 = AC^2 - BD^2$

$\therefore 2(AB(GA.BK) =$

$\therefore GA = BK.$



~~$AB^2 = AC^2 - AD^2 - DB^2$~~

$AD^2 = AC^2 + CD^2 + 2AC.CD$ (II 12)

~~$CB^2 = CD^2 + DB^2 + 2CD.DK.$~~

$CB^2 - AD^2 = AC^2 - DB^2$

$CB^2 - AC^2 = AD^2 - DB^2$

$CB^2 = CD^2 + DB^2 + 2CD.DK.$

$CB^2 - AD^2 = -AC^2 + 2CD(DK - AC)$

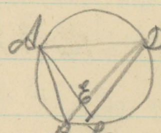
$CB^2 + DB^2 = AD^2 + AC^2$ $DK = AC$

$\cancel{CK^2} + KB^2 + \cancel{DK^2} + KB^2 =$

$AH^2 + \cancel{AD^2} + AH^2 + \cancel{AC^2}$

$2KB^2 = 2AB^2$

ex. prop. 27



∠AEB invar.

∠EAD invar. subtending CD

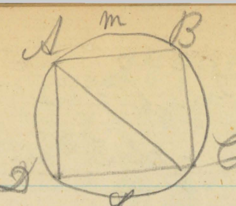
∠ADE invar. " AB

ex 2 prop 27

AB = CD ∴ ∠DAC = ∠ACB

∴ 31 I AB ∥ CD.

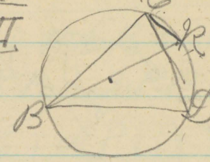
ex. 1 prop. 28



AB = DC chord ∴ arc AMB = DLC (III 28)

∴ ∠DCB = ∠DAC ∴ BC ∥ AD (I 37)

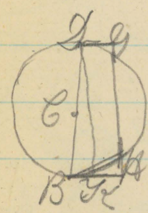
ex. 4 prop. 31 BK III



∠BCR rt ∠ in semi diff BCD & rt∠

= BCR but ∠BCR = rt∠

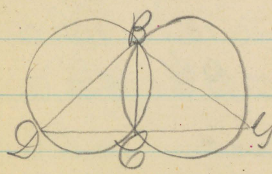
ex. 5



∠AEG = rt. ∠s ∠DKB rt∠.

∴ DK ∥ GH

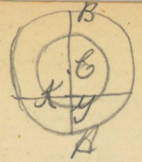
ex 6

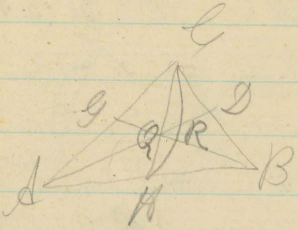


∠BCD rt. ∠ semi BCG rt. ∠ semi. circle

I 14 DCG st. line

ex. 7

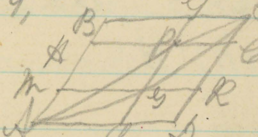




$\triangle AGB = \triangle ADB$ $\frac{1}{2}$ whole $\triangle$

$\triangle AQH = \triangle QHB =$

ex. p. 69,



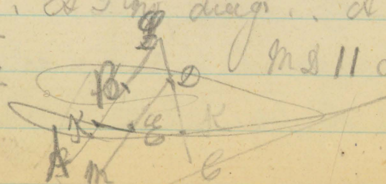
P pt. draw BC, GL join AP. PC

If AP. PC not st. line draw st. line AGC through G draw MR $\parallel$ BC

BD ▱ AGC diag. ▱ BG = GD comp. (I 43)

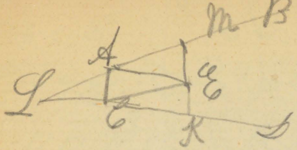
BP = GD ∴ AG diag. ∴ APC diag.

8 p. 56



MD $\parallel$ AF

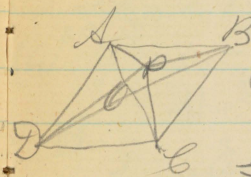
ex. 8 p. 56



AE $\parallel$ CD make AM = AL join BE to meet CD in K draw AC $\parallel$ MA

ex. 4 p. 72

diff. $\angle$ CPD & APB = sum. $\angle$ APC, BPD,



CPD + ~~PCB~~ = CDB + DCB

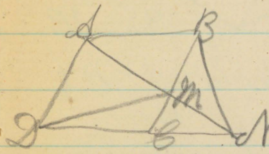
= ACB + DPB (I 37)

= APC + APB + ~~PCB~~ + DPB

∴ CPD − APB = APC + PBD.

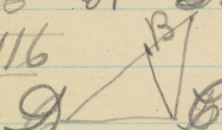
ex. 2 p. 69.

prove



F DB = DC join BC

ex. 3 p. 116

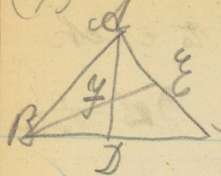


∴ $\angle$ FCD rt. ∴ $\angle$ D E F = 1 rt. $\angle$

D double F ∴ D = $\frac{2}{3}$ rt. $\angle$ F = $\frac{1}{3}$ rt. $\angle$

F & C = $\frac{4}{3}$ rt. $\angle$ ∴ $\angle$s B & C =

(4)



$AB^2 = 3AY^2$

(I A) $AE = EC$

$AB^2 = 4AE^2$

$AY = 2YE$

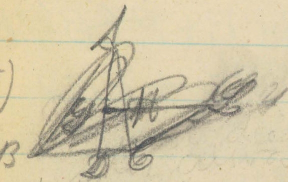
$AY^2 = 4YE^2$

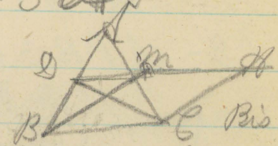
$AB^2 = 4AE^2 = 4AY^2 - 4YE^2$

$= 4AY^2 - AY^2$

$= 3AY^2$

(5)





Bis. AB

draw DM ∥ BC disc. ⊙ BC make MH = BC

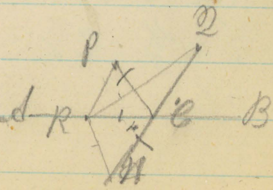
△ ADC = △ BDC (I 38)

▱ MC double △ BDC (I 41)

∴ ▱ MC = △ ABC MB = BC

∴ all sides equal.

(7)



∠ ACH = ∠ QCB = (I 15)

∠ PCR = QCB

PC = HC, RC com. ∠

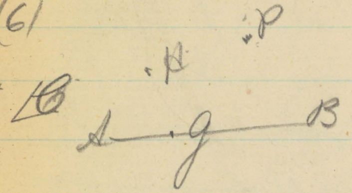
PR = HR (I 4) add QR to each

∴ PR, RH = QR, RH gr. QH than

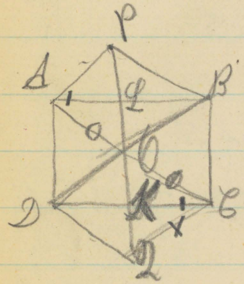
q. QC, CH, gr. PC. CH

∴ PR, QR g. PC, CH.

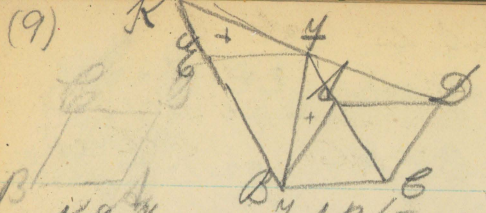
(6)



(8)



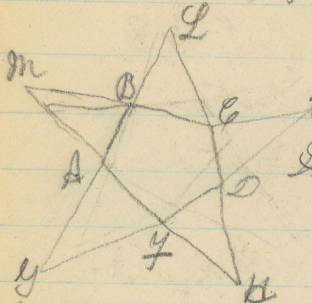
(9)



$\angle KEF = \angle FAB$ (I 29)

$EF = AD = $ Hyp.

$\angle KFE = \angle AFD$



(10)

K ~~Int = 6 rt. ∠~~

~~Int ∠s 4 rt. ∠s = 2~~

Int ∠s = 2n − 4 rt ∠s

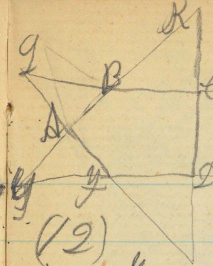
~~∠s of ext. Δ = 2n ∠s~~

all ∠s of Δs = 2n rt ∠s

∠s @ intersection = 2n − 8 rt ∠

(∵ ∠s @ bases = 8 rt ∠s

∴ ∠s pts. of inters. + 4 rt. ∠s = int. ∠s



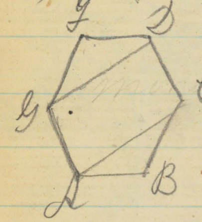
Int. ∠s = 6 rt. ∠s

R ext. Δs = 8 rt. ∠s

take away RCD & CDM

∴ ext. Δs = 6 rt. ∠s

(12)



∠s hex. = 8 rt. ∠

∴ each = $\frac{4}{3}$ rt ∠s

join GA, G D,

∠s int Δ = 2 rt. ∠s

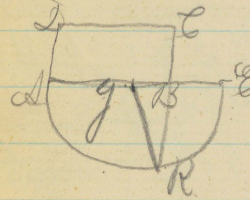
B = $\frac{2}{3}$ ∴ A & C = $\frac{2}{3}$ rt ∠

GAB & DCB = $\frac{8}{3}$ rt. ∠

∴ GAC, DCA = $\frac{4}{3}$ rt ∠s.

(∴ (I 27) GA ∥ DC)

(11)



rect. DB = sq BR

AB ½ perim. rect

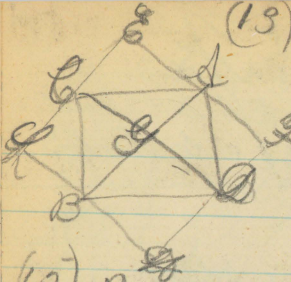
AG ¼ GR ¼

GR = BR = ¼ sq.

∴ ¼ perim rect. = ¼ perim. sq.

∴ perim. rect. = perim. sq.

(13)



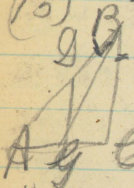
$\triangle CAD = \frac{1}{2} CD \times AG$

$\triangle CBD = \frac{1}{2} CD \times BG$

$\therefore ABCD = \frac{1}{2}(AG + BG) CD$

$= \frac{1}{2} CD^2$

(15)

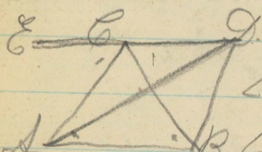


⊥ DG bis ∠ABC first

∠DBG = ∠GBC = ∠GBC

∴ DG = DB

(17)



∠ECA = ∠CAB (I 29)

∠CAB = ∠CBA

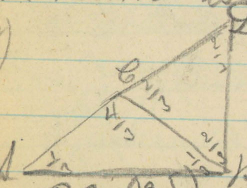
∠CBA = ∠DCB (29)

∴ ∠ECA = ∠DCB

∴ AC, CB less AD, BD (ex 7 p. 116)

∴ Perim. isos. △ < other △.

(18)



isos. △ACB ∠'s base ¼ vert.

$\therefore \angle ACB = \frac{4}{3}$ rt ∠

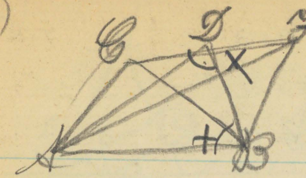
∠CBA, CAB = each $\frac{1}{3}$ rt ∠

$\angle \cancel{BCB}\ (13+) = \frac{2}{3}\ \angle D\cancel{BC} = \frac{2}{3}\ \therefore \angle \cancel{CDB}\ \frac{2}{3}$

~~7 DC, BD C 4/3 ∴ DCB = 2/3 rt ∠~~

∴ all ∠'s = ∴ △ equilat. (I A)

(16)

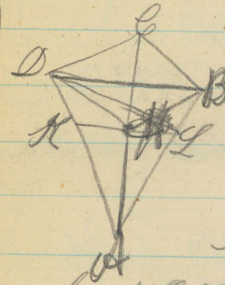


between same ∥ (I 39)

∴ CD7 = st. line

I ~~14~~ CD, D7 1 st line

(20)



draw CA bisect in H

∠CDH = ∠DAH & △CBH = △BHA ∴ quad bisected by DH & AB join BD.

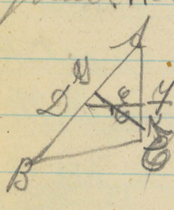
make KL through H join DL.

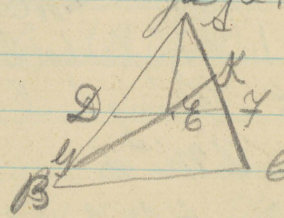
△DHB = △DLB (I 37)

add △DBC ∴ DBCL = pg DCBH

ax. 2. DCBH = ½ quad ∴ DCBL ½ quad ∴ bisected by D, L.

(19)



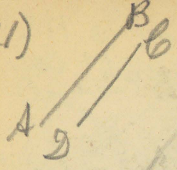


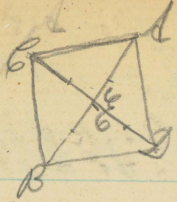
Suppose GK bis. in E join AE

△ADE = △AE7 I (38) △DGE = △AEK

I 38. △GED g △ADE ∴ g AE7 much g. △AEK ∴ GE not bisected in G.

(21)



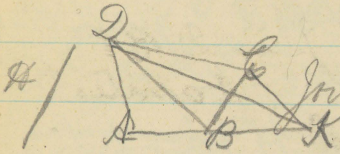


△ CEA, BED =, CA = BD

∠ CAE = ∠ EBD ∴ AC ∥ BD

Sim. shewn CB ∥ AD ∴ rhombos.

(22)

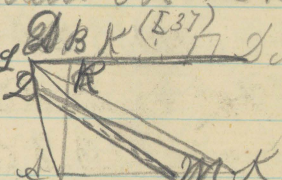


Join DB prod AB

through C draw CK ∥ DB join DK

△ DBC = △ DBK (I 37) ∴ ▱ DABC = △ ADK

(put △ alone)



draw ~~AL~~ ⊥ AK prod AD to meet KL

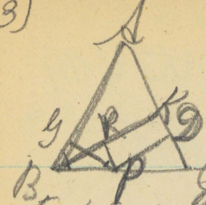
join LK through D draw DM ∥ LK. join LM.

△ LDM = △ KMD I 37 add DAM

∴ △ DAK = △ LAM ∴ △ drawn

as reqd. LMA = ▱ ABCD alt = A.

(23)



Join PR ∥ AD

PD = RK (I 34) In △s

BGP, RBP, BP comm

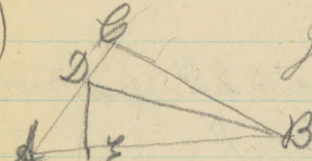
∠ BGP = ∠ BKP = both

rt. ∠s, ∠ GBP = ∠ RPB ∴ △ = ∴ GB =

PR, BR = GP ∴ sum BP, PK = sum GP PD

∴ BK = sum GP, PD

(24)



Join DB

∠ A acute

(II 13)

$DB^2 = DA^2 + AB^2 - 2 AB \cdot AE$

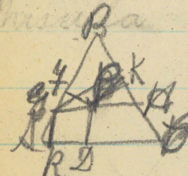
$AB^2 = AD^2 + DB^2 + 2 AD \cdot DC$ (II 12)

~~$DB^2 = AD^2 + DB^2 + 2 AD \cdot AC - 2 AE \cdot AB$~~

$AE \cdot AB = AD^2 + AD \cdot DC$

$= AC \cdot AD$ (II 13)

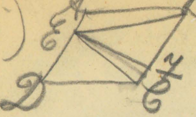
Max/ma



sum ⊥s constant

+ P E PK + PD

Join do

(25)  EF bis. ▱ ABCD

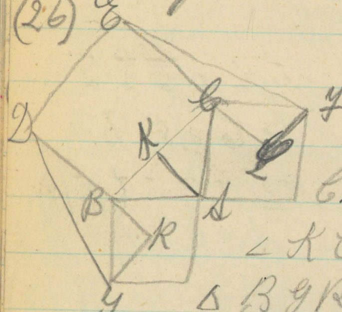
△ EBC = $\frac{1}{2}$ ▱ (I 41)

fig. EDCF = $\frac{1}{2}$ ▱ by hyp.

∴ EDCF = △ EBC

Take away com △ EFC ∴ △ EDC = △ BEF.

(26)



△ ELF = △ KCA

CA = CF ∠s L & K rt.

∠ KCL = CKA sim,

△ BGR = △ BKA,

CL = CK | $EF^2 = EL^2 + LF^2$ (I 47)

$EL^2 = EC^2 + CL^2 + 2EC.CL$ (II 4)

$LF^2 = CF^2 - CL^2$

$EC^2 = BC^2$ CL = CK ∴

$EL = BC^2 + 2BC.CK$

$LF^2 = AC^2$

∴ $EF^2 = AC^2 + CB^2 + 2BC.CK$

$2BC.CK + 2BC.BK = 2BC^2$ (II 2)

$GY^2 = EC^2 + CY^2 + 2EC.CL$

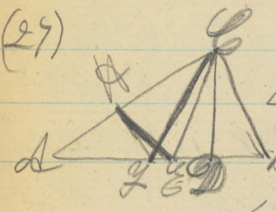
$= BC^2 + CA^2 + 2BC.CK$

$DG^2 = BC^2 + BA^2 + 2BC.CK$

$EF^2 + DG^2 = 2BC^2 + BC^2 + 2BC^2$

$= 5BC^2$

(27)

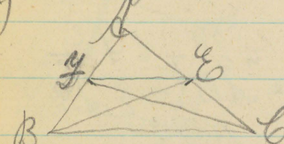


∠ CDE rt.∠ ∠ CED less ∴ EC g CD.

∠ CEF g rt∠ ∴ CFE less

∴ FC g EC.

(28)



$4BE^2 + 4CF^2 = 5BC^2$

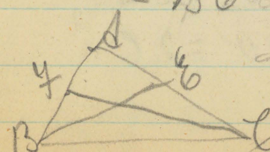
$BE^2 = BA^2 + AE^2$

$FC^2 = AC^2 + AF^2$

$BE^2 + CF^2 = BA^2 + AC^2 +$

$= BC^2$

(28)



$AB^2 + BC^2 = 2BE^2 + 2AE^2$

$AC^2 + CB^2 = 2CE^2 + 2CF^2$

$3CB^2 = 2BE^2 + 2AC^2 + 2CF^2 + 2CE^2$

$2CF^2 + 2BE^2 = 3CB^2 - 2AE^2 - 2CE^2$

$4(CF^2 + BE^2) = CB^2 - 4AE^2 - 4CE^2$

$= 6BC^2 - BC^2$

$= 5BC^2$ ans.

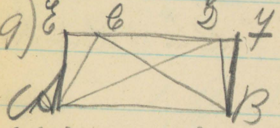
(28) $4BE^2 + 4CF^2 = 4AB^2 + 4AE^2$

$+ 4AF^2 + 4AC^2$

Best way $= 4BC^2 + AC^2 + AB^2$

$= 5BC^2$

(29)



$CE^2 = CA^2 - EA^2$

$CF^2 = CB^2 - DF^2$

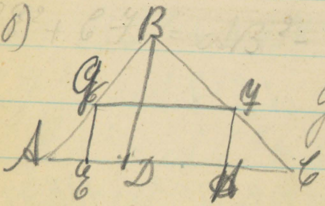
$CE^2 + CF^2 =$
$AB^2 - EA^2 - BF^2$

$DE^2 = AD^2 - AE^2$

$DF^2 = AB^2 - DF^2$

$DE^2 + DF^2 =$
$AB^2 - AE^2 - BF^2$

(30)



$GF \parallel AC$ & $= \frac{1}{2} AC$

$AE\ \frac{1}{2} AD$

$DH\ \frac{1}{2} DC$

$\therefore EH\ \frac{1}{2} AC$

$\therefore GF\ \frac{1}{2} AC$

(31)

$BD.DC = AB^2 - AD^2$

$AB^2 = AK^2 + KB^2$

$AD^2 = AK^2 + KD^2$

$AB^2 - AD^2 = KB^2 - KD^2$

$= BD.DC.$

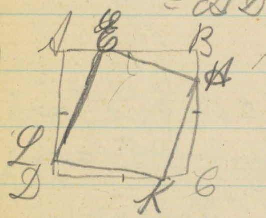
Another Way.

$AB^2 = AD^2 + DB^2 + 2BD.DK$

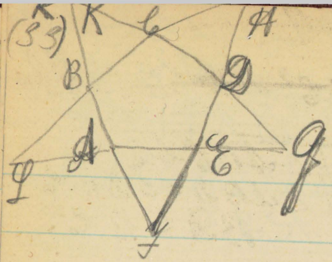
$= AD^2 + DB^2 + BD.DK$
$+ BD.KD$

$= AD^2 + KC.BD + BD.KD$

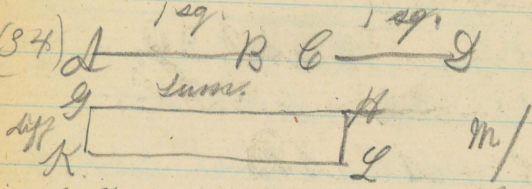
(32)



(33)



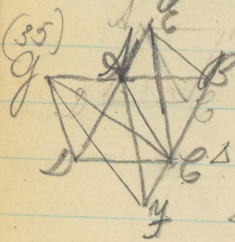
(34)



GL = rect. sum & diff. AB^2, CD^2

rect $GL = M^2$ $\therefore$ M^2 = diff. AB^2 & CD^2

(35)



$\Delta AEB = \Delta AEC$ join CG

~~$\Delta ACY = \Delta GAC$ (I 37)~~

$\Delta EAC = \Delta ACY$ (I 37)

$\Delta ACY = \Delta CGA$ (37)

$AGC = GAD$ (37)

(36)

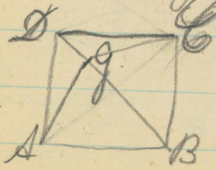
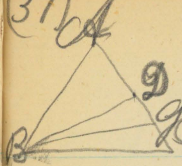


fig. $DAYC = 2\,DG^2$

fig. $= \frac{1}{4} DB^2$

$DG^2 = \frac{1}{16} DB^2$

(37)



~~$BG^2 + AC$.~~

~~$BA^2 = BC^2 + AC^2 - 2\,AC$~~

~~GC.~~

~~$BC^2 = 2\,AC.CG$.~~

~~$= AC.CD$~~

$\angle ABC$ bis.

$\angle ABD = \angle DBC$

$\therefore BD = BC$

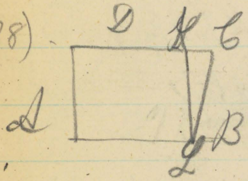
$BA^2 = BD^2 + DA^2 + 2\,AD.DG$.

$= BC^2 + AD^2 + AD.BC$.

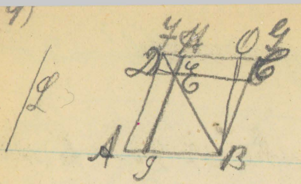
$= BC^2 + AC.AD$

$= BC^2 + AB.BC$,

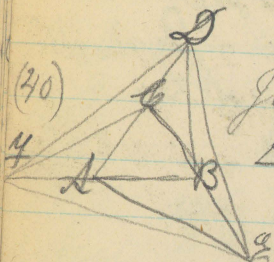
(38)



(39)

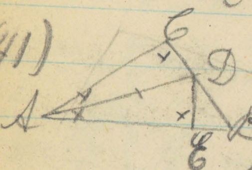


(40)



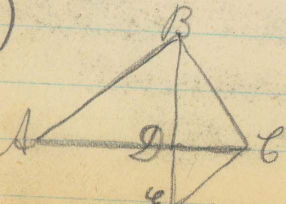
Join CF, AE, BD,
Δ all equal. by
(38 I)

(41)



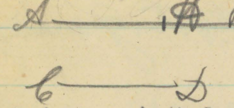
In Δs ACD; ADE
$CD = DE$ (I 26)
$\angle DEB = 1$ rt $\angle$,
$\angle DBE = \frac{1}{2}$ " "
$\therefore \angle EDB = \frac{1}{2}$ " "
$\therefore DE = EB$ (I A) sq. $DB = DE^2 + EB^2$,
(I 47) $\therefore DB^2 = 2DE^2 \therefore = 2CD^2$

(42)



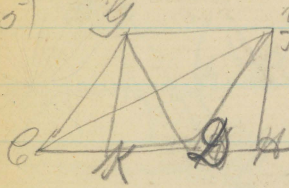
$BC^2 = BD^2 + DC^2$
$= AD.DC + BD.DE.$

(43)



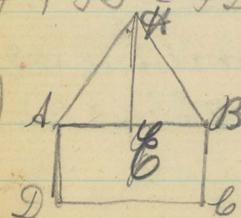
Make $AH = CD$
then $AB = AH + HB$
$AB^2 = AH^2 + HB^2 + 2AH.HB$
$AB^2 + CD^2 = 2AH^2 + 2AH.HB + HB^2$
$= 2AB.AH + HB^2$ (II 3)
$= 2AB.CD + HB^2$

(40)



$CF^2 = FD^2 + DC^2 \cancel{+ 2CD.DH}$
$FG^2 = CG^2 + CD^2 - 2CD.CK$
$CF^2 + GD^2 = FD^2 + CG^2 + DC^2 + GF^2$

(46)



$HG \perp AB$
Area $AC = AB \times BC$
$\cancel{= \frac{1}{2}AB^2 = \frac{1}{2}AH^2}$
~~but $AH = HG$.~~
$= AB \times AE$

area $\triangle AAB = AB \times \cancel{AE}\,\mathcal{E}$

$A\mathcal{E}$ less AB $\therefore$ $AB \times A\mathcal{E}$

g $AB \times A\mathcal{E}$ $\therefore$ area rect. g area. $\triangle$

(47)

$AB^2 = AO^2 + OB^2 +$

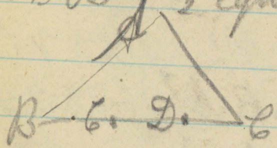
$2 AO \cdot OD$

$\wedge \because \angle$ around $O = 4$ rt. OB double OD

$\angle AOB = \frac{1}{3}$ of 4 rt $\angle \wedge = \frac{2}{3}$ 2 rt $\angle s$

$\angle BOD = \frac{1}{3}$ 2 rt. $\angle$. i.e. $1\angle$ of equilat. $\triangle$

$\therefore BOD = \frac{1}{2}$ equilat. $\triangle$



(49)

red

funny

$BD^2 + C\mathcal{E}^2 = BC^2$

$= B\mathcal{E}^2 + \mathcal{E}D^2 + DC^2$

$+ 2B\mathcal{E}.\mathcal{E}D + 2B\mathcal{E}.\mathcal{E}C + 2\mathcal{E}D.DC$

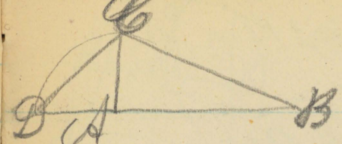
$BC^2 = B\mathcal{E}^2 + 2\mathcal{E}D^2 + DC^2 + 2B\mathcal{E}.\mathcal{E}D$

$+ 2\mathcal{E}D.DC$ (II 4)

$= 2B\mathcal{E}.CD - \mathcal{E}D^2$

$\mathcal{E}D^2 = 2B\mathcal{E}.CD$

(50) A———B side rect. | 8



(II 12)

(47) con. $AB^2 = AO^2 + OB^2 + 2AO.OD$

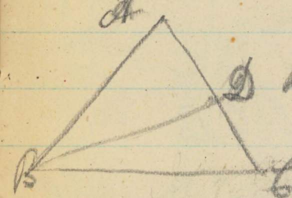
$= AO^2 + OB^2 + AO.OB$.

$BC^2 = BO^2 + OC^2 + OB.OC$.

$AC^2 = AO^2 + OC^2 + AO.OC$

$AB^2 + BC^2 + OC^2 = 2AO^2 + 2BO^2$

$+ 2CO^2 + AO^2 + CO^2 + BO^2$



$BD^2 = BA^2 - AD^2$ (I 47)

$= AC^2 - AD^2$

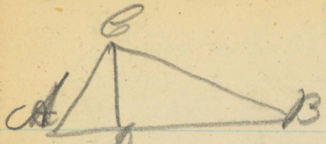
$= \cancel{AD^2} + DC^2 + 2AD.DC - \cancel{AD^2}$

$= DC^2 + 2AD.DC$

he went too far g. 1 examin

d. r

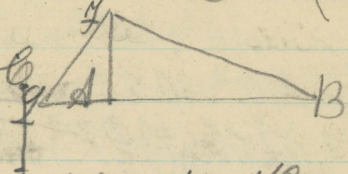
1 p. 93



$AB \cdot AD = AD^2 + \cancel{AD} \cdot DB.$

$= AD^2 + DC^2$ (ex 4. II)

8 p. 93.

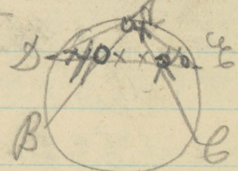


ex II 4 $AF^2 = AB \cdot AL$,

~~9~~ (93)] A C D B

$AB^2 = AC^2 + CB^2 + 2AC \cdot CB$

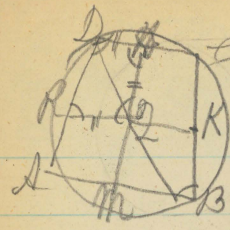
ex 4 p. 164



$\angle DAE = \angle AEC$ subt. by AD same arc

$\angle ADE = \angle CAE$ ∴

$\angle AHD = \angle AKE$ ∴ $AH = AK$



$\angle RDH$ rt. $\angle$ (4)

$\angle DHQ$ " "

$\angle DRQ$ " "

∴ $\angle RQH$ rt. $\angle$ ∴

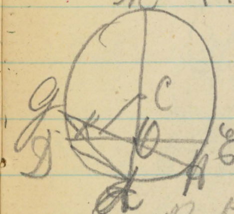
QD rect. ∴ $QR^2 = DH^2 + HQ^2 = HQ^2 + QR^2$

$\angle MBC$ rt. $\angle$

QKB rt. $\angle$ ∴ MQK rt. $\angle$, rect.

∴ $QB^2 = QK^2 + KB^2 = QK^2 + MQ^2$

∴ $QD^2 + QB^2 =$ sqq. on perpen.



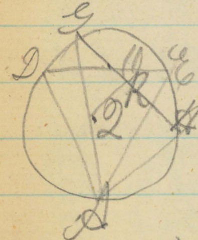
draw ⊥ CK

chord GH g ch. DE

∴ seg GHg seg DE

But ∠ in greater seg. less

∴ ∠ in seg DE g. than ∠ GH.

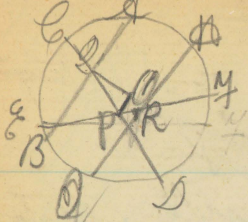


seg. GHg seg DE

∠ subtends by g seg g. ∠ sub

less seg. ∴ ∠ g.

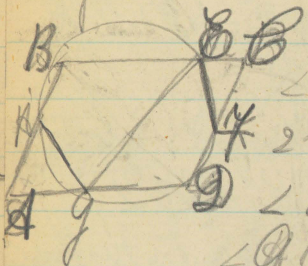
(5)



$\angle OPC$ rt $\angle$ $\therefore$

$\angle OQP$ less " "

$\therefore OQ$ g. OP.



(8)

$\angle GEF$ & $\angle GDF$ = 2 rt. $\angle$s. (22 III)

$\angle BGE$ & $\angle GBE$ = 2 rt $\angle$

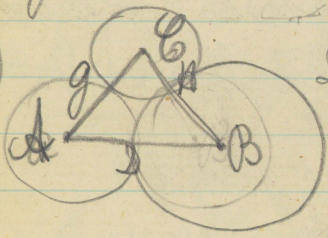
$\angle GEF = \angle GDE$

$\therefore \angle ABC = \angle GDE$ (I 34)

$\therefore \angle AGE = \angle GEF$ $\therefore$

$BG \parallel EF$ (I 27)

(9)

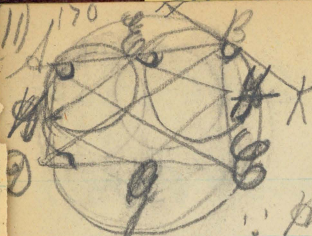


Diff. between AC
CB = diff. AG
$GA \therefore GC = CA$

diff. AG, HB same as diff AD, DB

" A & B invar $\therefore$ diff. AG CB inv

11) 170



$EBGF = \square$ $\therefore$

opp. sides = draw tangent

Join AC, BD.

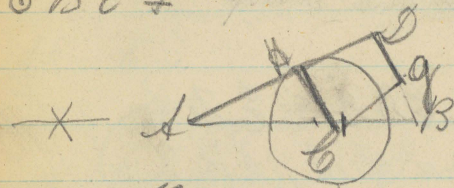
$\therefore HK$ tangent $\therefore \angle KBC$

$= \angle BDC$ & $\angle BDC = \angle BAC$ same seg.

$EF \parallel AC$ $\therefore \angle BEF = \angle EAC$

$\therefore \angle BEF = \angle KBC$ $\therefore$ HK tangent

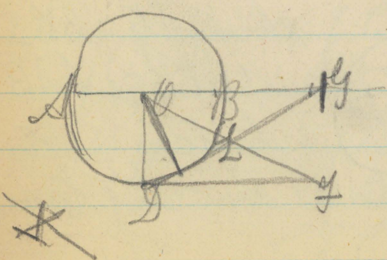
$\odot BEF$



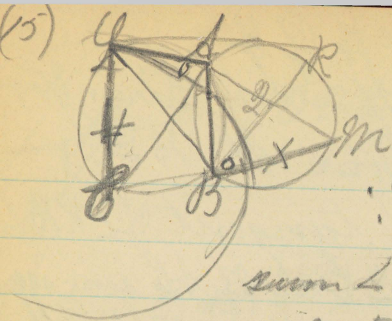
$CH \parallel DG$.

$\because DG \parallel CH$ & HD joins $\therefore \angle H = \angle D$

$\angle D$ rt. $\angle$ $\therefore \angle H$ rt. $\angle$ $\therefore AD$ tan 16 III



(15)



BM = CL

∠ LAC = ∠ ABM

∠ 37

∴ arc LC = BM

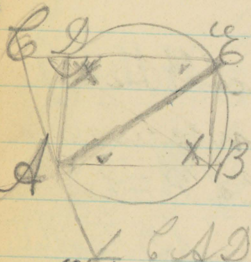
sum ∠ ABM, ABC

= 2 rt. ∠s

2nd part

Join KL

(16)



CDA, & EDA = 2 rt. ∠

EDA, ABE = "

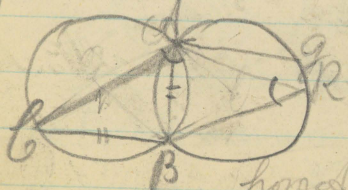
∴ ∠ CDA = ∠ ABE

∠ CAD = ∠ DEA ∴ tan.

~~EAB~~ = ∠ EAB.

∠ ACD = ∠ AEB.

(17)



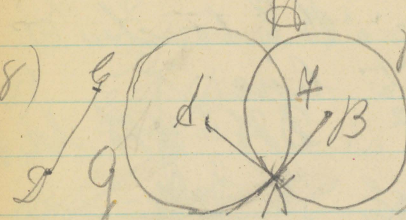
chord AB = BC

∴ ∠ ARB = ∠ CAB

AC tan. if not

prove drew AG tan.

(18)



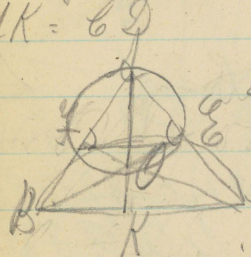
∴ ∠ AKB

rt. ∠ ∴ AK

tan. ⊙ KHR

AK = CD

(19)



∠ AFO & AEO = 2 rt. ∠

∴ ⊙ can be desc.

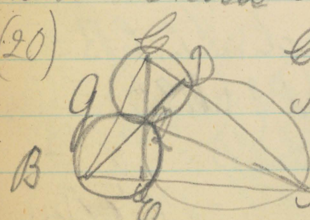
∠ BFC & BEC, 2 rt.

∴ ⊙ desc. about

Join FE ∠ FAO = ∠ FEO sub by

same chord ∠ FEO = ∠ FCB = FAO

(20)



CE.CQ = ~~CB.CG~~

AG.AQ = AB.AE

~~BD.BQ = BG.BC~~

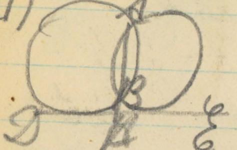
AG.AQ = AC.AD

CE.CQ = CA.CD

BD.BQ = BA.AE

(side) $= 2CE.CD + 2AG.AD + 2BD.B$
$= BC^2 + AB^2 + AC^2$

(21)



$AB.BD = DH^2$

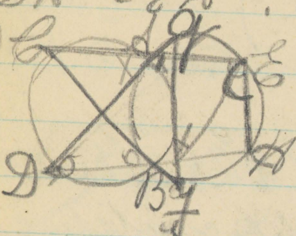
$'' \quad '' = HE^2$

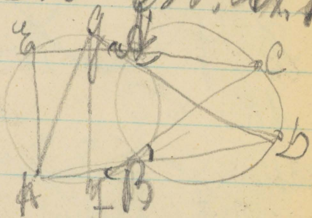
∴ HE touches ⊙

and AH cuts line.

∴ $DH^2 = EH^2$ ∴ $DH = EH$. AH.

(22)



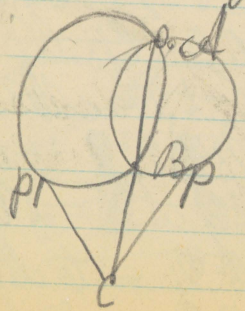


∠CAD = CBD (III 21) ∠EAG = ∠CAD

∠EAG = ∠HBF = ∠HGF

∠HGF = ∠EHG alt. ∠ ∴ EH ∥ GF

(23)

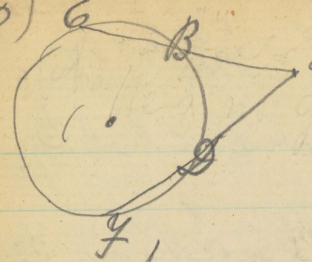


rect. AC.CB = CP^2 (III 36)

$AC.CB = CP'^2$,

∴ PC = PC' ax I 46

(25)



rect AE.AB

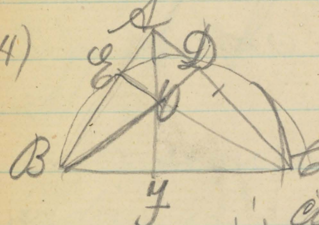
= '' AD.AF

AB = AD

∴ EB = FD

∴ equal dist. ⊙

(24)

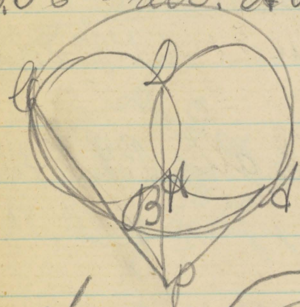


∠BEC = 1 rt. ∠

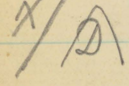
∠BDC = '' ''

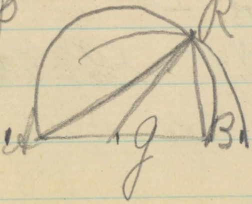
∴ circle desc. on BC passes through E & D. ∴ rect. DO.OC = rect. AO.OF = rect.

(27)



(28) X / D

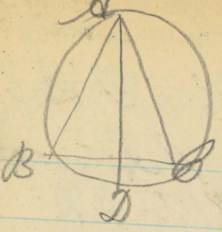




desc. seg. g. semi. ∴ ∠ acute

∴ R = ∠D and RG = X

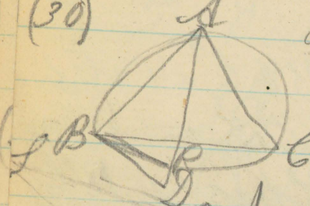
(29)



$\angle BCD = \angle BAD$

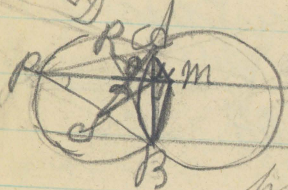
$\angle BAD = \frac{1}{2} BAC \therefore \angle BCD = \frac{1}{2}\angle B$

(30)



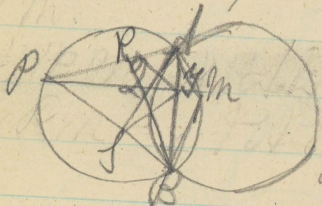
disc. ⊙ touch. A B, C

(31)



Prod. PQT to M
join AM & BM
prove $\angle APM = \angle$
BPM.

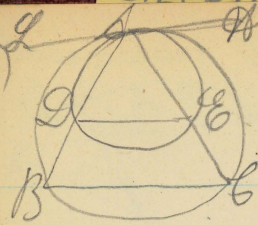
(31)



Prod PQT. to M
join AM, MB

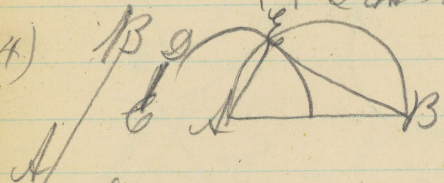
Prove $\angle QRYP = \angle PTS$

(33)



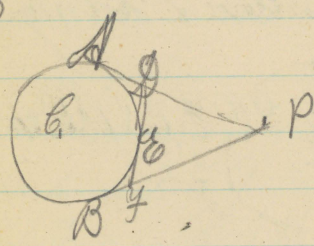
$\angle HAC = \angle ADC$
$ABC = ADE$
$\therefore \angle HAE = \angle ABE$
$\therefore$ LH tan.

(34)

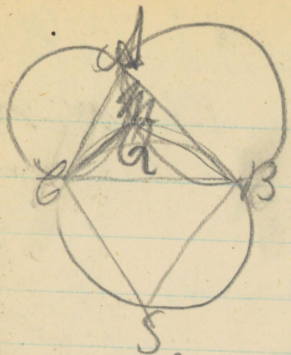


make semi. ⊙ on AB make ⊙
radius = CD. Join AE, EB
$\angle AEB$ rt. $\angle$. $\because \angle$ in semi. ⊙
$AE^2 = CD^2 \therefore$ diff. AE^2 & $AB^2 =$
EB^2

(35)

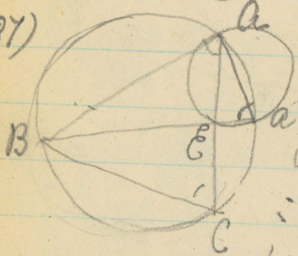


(36)



Let ~~intersection~~ of AQC in M

(37)



$BA = BC$

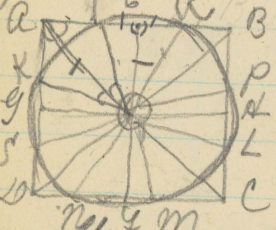
$\therefore \angle BAC = \angle BCA$

$ACB = AC'B$ III 2

$\therefore \angle AA'E = BAC$

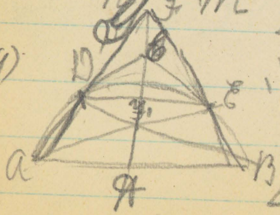
$\therefore BA$ = tangent

(38)



GO, EO, OH each rt $\angle$

(39)



$\angle FDG$ & FEG rt

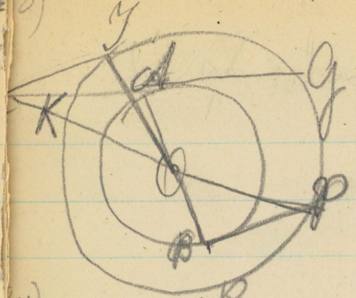
$\therefore \odot$ ~~C~~ EFD. ΔGDF

GHA. $\angle BGF$ com.

$\angle GDF = \angle GHA$

rem $\angle DFG$ = rem GAH

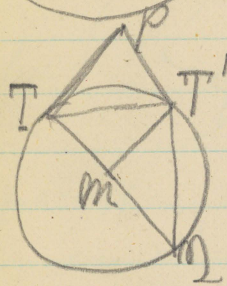
(40)



$OP^2 = OB^2 + BP^2$

$\therefore BP^2 =$ diff. OP^2 & OB^2 = sqq. on radii

(41)



$\angle PYY' = \angle PY'Y$

$\angle YPY' =$ rt. $\angle$

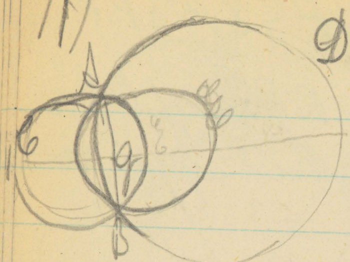
(39) but $\angle DFG = CDF \therefore \angle$

$CDF = \angle GAH$ ext = $\angle$ in

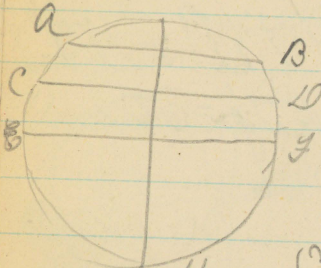
opp. seg. $\therefore DC$ tan. $\angle$ C E tan.

loci

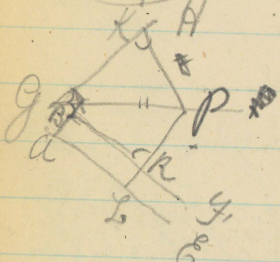
(1)



(2)



(3)

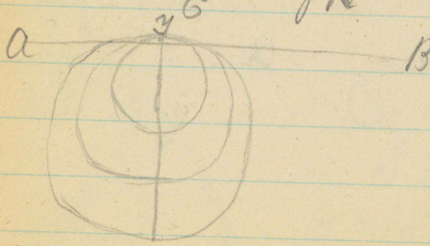


$\angle KGP = \angle FGP$

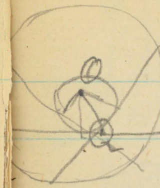
∵ bis. $\angle GP$ com $\angle GKP$

$\angle GRP$ rt∠ ∴ $KP = PR$

(4)

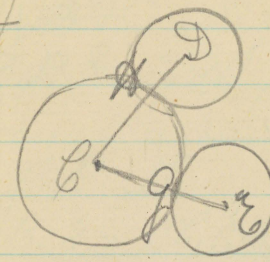


(5)



Locus = ⊙ desc. on OQ

(6)



$CG = CH$

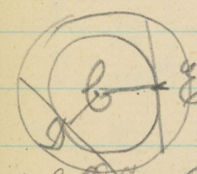
$HD = GE$ (hyp.)

∴ △s = ∴ $CE = CD$

⊙ cen. C rad CE

pass through D E. Locus = ⊙ desc. with sum radii of given desc ⊙

(7)

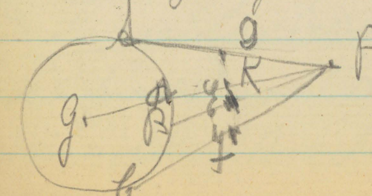


equal chords equally dist. from centre. (15 III.)

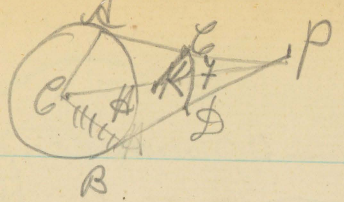
∴ $CD = CE$. ∵ $DC \perp CE$ ∴ ⊙ desc. cen. C dis DC passes through D & E

∴ Locus = circle with cen. of given ⊙ & line joining middle pt. of chord

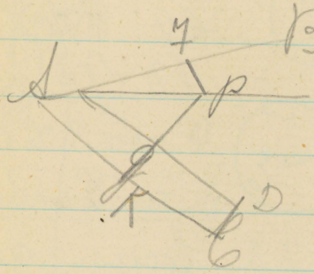
(8)



(8)



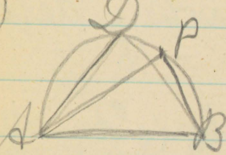
(3)



Perpu. CD - D

1D

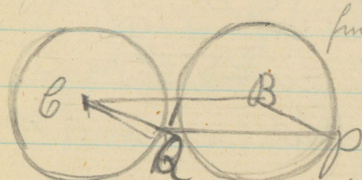
(43)



locus = ⊙ disc, ~~line~~ AB and con

$\angle$ = given $\angle$.

(44)



find centre C

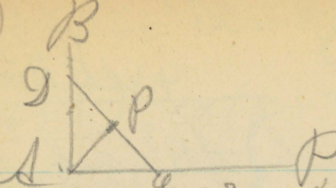
CB $\parallel$ QP & =

Join CQ, BP

$\therefore$ CQ = BP

CQ constant $\therefore$ BP constant $\therefore$ P moves in a ⊙ $\therefore$ locus of P = ⊙

(45)



$AD^2 + 2AE^2$

$2AP^2 + \cancel{2EP^2} = DE^2 = 4DP$

$= 2DP^2 \therefore$ A fixed pt.

$\therefore$ locus = ⊙ disc cent. A dist. AP.

BK II

I $AB.E = AC.E + CD.E + DB.E$

A C D B

E Let $AC = a$ $CD = b$ $DB = c$

$E = d$

$AB = a + b + c$

rect. $AB.E = (a+b+c)d$

Algebraical Proof $= ad + bd + cd$

$= AC.E + CD.E + DB.E$

Prop. II

Straight line

A C B $AB^2 = AB.AC + AB.CB$

$AB^2 = AB.AB = AC.AB + CB.AB$ (II 1)

A C B $\quad AB^2 = AB.AC + AB.BC$

$AC = a \quad CB = b$ $\Big\}$ $AB^2 = (a+b)^2 = a^2+2ab+b^2$

$AB = a + b$

(Algebraic Proof)

$= (a^2+ab)+(b^2+ab)$

$= a(a+b)+b(a+b)$

3 Prop. $\quad = AC.AB + CB.AB$

A C B $\quad AB.CB = AC.CB + CB^2$

$AB.BC = \text{rect. } AC.CB + CB.CB$ (II

$= AC.CB + CB^2$

Algebraic. A C B

$AC = x$

$CB = y$

$AB = (x+y)$

$AB.CB = (x+y)\,y$

$= xy + y^2$

$= AC.CB + CB^2$

(1) $a(b+c+d+\cancel{dc}) = ab+ac+dc$

(2) $(a+b)^2 = a(a+b) + b(a+b)$

(3) $a(a+b) = a^2+ab$

(4) $(a+b)^2 = (a^2+2ab+b^2)$

A C B $\quad AB^2 = AC^2 + CB^2 + 2AC.CB$

$AB^2 = AB.AC + AB.CB$ (II 2)

$= AC.CB +$

(Algebraic)

$AC = a \qquad AB^2 = (a+b)^2$

$AB = b \qquad = a^2+2ab+b^2$

$\cancel{Ab} = (a+b) \qquad = AB^2 + 2AC.CB + CB^2$

A C D B

$CB^2 = CD^2 + DB^2 + 2CD.DB$

$= CD^2 + DB^2 + CD.DB + CD.DB$

$= CD^2 + CB.DB + CD.DB$ II 3

$= CD^2 + AC \cdot DB + CD \cdot DB$

$= CD^2 + AD \cdot DB$ (II 1)

A — C D B

Alg. $AC = a$

$CD = b$

$AD = (a+b)$

$BC = a - b$

$DB = (a-b)$

$AD \cdot DB + \cancel{CD}^2 =$

$(a+b)(a-b) + b^2$

$= a^2 - b^2 + b^2$

$= a^2$

$= \cancel{C}B^2$

A — B C D $\cancel{AD \cdot DC + BC^2} = BD^2$

$= CD^2 = CB^2 + BD^2 + 2 BC \cdot BD$ II 4

$= CB^2 + BD^2 + CB \cdot BD + \cancel{CB} \cdot BD$

$= CB^2 + CD \cdot BD + AC \cdot BD +$ II 3

$= CB^2 + AD \cdot BD$

If a st. line be equally & unequally whether internally or externally the rectangle contained by the unequal segments = diff. of sqq. on half line and line between pts. of ~~section~~ segments

B A D B C prod to CA

$AC \cdot AD = AB^2 - BC^2$

$CA \cdot AD + BC^2 = AB^2$ II 4

$AC \cdot AD = AB^2 - BC^2$

(7)

A — C B

$AB^2 + AC^2 = 2 AB \cdot AC + CB^2$

$= 2 AC^2 + CB^2 + 2 AC \cdot CB$ II 4

$= CB^2 + 2 AB \cdot AC$

$AB^2 + AC^2$

9 A C D B

$AD^2 + DB^2 = AC^2 + CD^2 + 2AC \cdot CD$
$+ DB^2$

$= AC^2 + CD^2 + 2CB \cdot CD$
$+ DB^2$

$= AC^2 + CD^2 + CB^2 + CD^2$
$= 2AC^2 + 2CD^2$

$AC = a$
$CD = b$
$AD = a + b$
$DB = a - b$

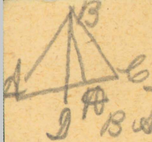
$AD^2 + DB^2 = (a+b)^2 + (a-b)^2$
$= a^2 + 2ab + b^2 + a^2 - 2ab + b^2$

A C D B $= 2a^2$

(10) A C B D

$AD^2 + DB^2 = 2AC^2 + 2CD^2$
$= AC^2 + CD^2 + 2AC \cdot CD$
$+ 2BD^2$ ~~Z~~ 4

$= AC^2 + CD^2 + 2CB \cdot CD + \cancel{2}BD^2$
$= AC^2 +$



$$AB^2+BC^2=2AD^2+2DB^2$$

$$BA^2=AD^2+DB^2+2AD\cdot DA\cdot(\text{II}/2)$$

$$BC^2=BD^2+DC^2-2DC\cdot DA\cdot(\text{II}/3)$$

$$BA^2+BC^2=2BD^2+2AD^2$$

BRITISH ASSOCIATION.

Monday.	9—Breakfast—Adelphi Hotel. 10—Sectional Committees. 11—Sections. 5—Ordinary—Lucas's Room. 8—General Meeting—Amphitheatre.
Tuesday.	9—Breakfast—Adelphi. 10—Sectional Committees. 11—Sections. 5—Ordinary—Lucas's Room. 8—Soireé—Town Hall.
Wednesday. ..	9—Breakfast—Adelphi. 10—Sectional Committees. 11—Sections. 5—Ordinary—Lucas's Room. 8—Lecture—Amphitheatre.
Thursday.	9—Breakfast—Adelphi. 10—Sectional Committees. 11—Sections. 3—General Committee for appointing next Meeting. 5—Ordinary—Lucas's Room. 8—Soireé—Town Hall.
Friday.	9—Breakfast—Adelphi. 10—Sectional Committees. 11—Sections. 3—Dejeunè—Botanic Garden. 5—Ordinary—Adelphi. 7—Liverpool Academy.
Saturday.	9—Breakfast—Adelphi. 1—General Committee for Reports, Recommendations, &c. 5—Ordinary—Lucas's Room. —General Meeting—Amphitheatre.