

Dynamics.

of the distinguishing properties of matter —

Philosophers have been at great pains in endeavouring to ascertain in what consists the essence of matter, or of that substance of which all body is composed. But their want of success has arisen from their inaccurate Idea of the object of their search — If by the nature or essence of things we mean that which constitutes with the thing & which renders it that thing & no other or is the principle from which all its properties flow, & rendering it susceptible of all its modifications to which it is subject it follows that the essence of every thing is peculiar to itself & common to it is no other being in the universe — every body has its own essence — If we take it in this sense, it is evident that we are ignorant of the essence of matter. we do not know the principle which constitutes the essence of one body as distinguished from another. a stone from water much less do we know the circumstance which is the original of all the common properties of matter, distinguishing it from what ever is not material — The essence of a thing must be totally distinct from any property of that thing however general may the

universal. what we call it property of a thing is
reference to some other thing, & the name which we give
it is only an expression of an event. When we say
that gravity is a property of matter it is only a
short way of expressing the event that every body
will fall when not supported.

But if absence of matter must be competent to it
independent of every other thing & must refer only to itself
Not only so but it must be peculiar to matter, belonging
to nothing else; farther, it must be the cause of all it
properties of *M^r*. But our knowledge does not yet
reach so far. we cannot discover it manner in which
the different properties of *m^r* exist in one substance, &
what is the bond it unites them - our knowledge is con-
fined because in the examination of *m^r* we proceed on
the information of our senses; these give us our ultimate
principle, & yet by their means we can never penetrate
beyond it surface of things as to it constitution & nature
of an atom we are entirely ignorant. Nor are we
conceive a notion of an atom by what we observe in
the mass - an atom of Gold cannot be ductile nor can
not dissolve in aqua regia - the atom of an animal mus-
cular fibre cannot be irritable we can know only
those properties which affect our senses, & it is not at all
necessary if these properties should do so.

This what we formerly delivered in respect to
it & effect all our meaning by it term power is
only an expression of a connection it subsists in the
mind between two events. But did we know it absence
of matter we would be perfectly acquainted with every
cause. Upon the whole, all it we can do is to discover
the properties of matter & to arrange them according to
their various generalities, taking it for granted that those
it are more general & nearest allied to it essential, &
first properties of matter accordingly. Some philoso-
phers proceeding in this manner ^{matter} found it after all pos-
sible abstractions have been made that extension is it ge-
neral property of *m^r* being competent to all matter, &
distinguishing it from every other kind of being, & they
have even attempted to shew it from it flowed every
other property, & therefore that extension constituted it
absence of matter.

But this doctrine is ill founded, hasty, and such an
abuse of common language it entirely destroys its mean-
ing. all can, and do conceive extensions without any *m^r*
and space or vacuity it is nothing but extension without *m^r*
extension then is not peculiar to *m^r* and is common to it to
space, & all that is commonly said about the infinite
divisibility of *m^r* relate only to space, it object of geo-
metry but by no means of nat. philosophy.

Others have placed the essence of matter in motion.
But it is evident from Geometry that this may be
communicated without matter; and such motions are every
day perceived by us. no one can say that the shadow of a
sun-dial is *m^r*. tho' by its motion we compute of time.
no one can deny the mobility of the thinking
principle of animals, when he sees it always accom-
panying the body. nor let any man say that these
motions are the consequences of the motions of *m^r*. they
are distinguishable from the motions of *m^r*. & there-
fore separable from them.

The fact is that not only we shall & must
remain ignorant of the nature of *m^r*. so long as all our
information comes from experience, but these phi-
losophers have taken the improper method for attain-
ing the notions of *m^r*. w^{ch} we may acquire.

It is not by abstractions that we have acquired the
distinctive notions of any substance. It is by
observation, where we see the substance in its in-
dividual & complex state vested with all its qualities.
I therefore pay little regard to all that refinement
of philosophy has discovered, and in no case allow them
to have concurred in giving us any of the usefull knowledge
of different subjects w^{ch} is acquired by all men. I am incli-
ned to do the same thing here, & assign it as the original
of our distinction that is to say I consider these qualities

matter hinders any other *m^r*. from occupying
the same place w^{ch} itself as constituting the distinctive
property of matter; but if same time acknowledging
that I do not know any thing of its essence.

Solidity is an universal property of *m^r*. and completely
distinguishes it from spirit, and whatever is not material
whatever has this property is considered by us as matter.

Mobility belongs to it in common wth other things, &
is not essentially necessary; for there is no absurdity or
contradictions in supposing that *m^r*. must be absolutely
immoveable; the power of communicating motion only
belongs to *m^r*. when in motion and is therefore not
competent to it at all times. Even extension belongs
to it only accidentally, for if some, we admit the divisi-
bility of *m^r*. without end, and thro' infinite of infinitesimal
orders, we must conceive the ultimate object of our contem-
plation as unextended. In short, without any farther
enumeration, every other quality which has been as-
cribed to *m^r*. may be taken from it; but strip it of
solidity & we destroy the very Idea, & compound it to
space or nonentity.

But let us form just notions of the quality w^{ch} I thus
assert to be the distinctive property of matter.

It is quite distinct from the solidity of
for he talks of prisms standing on the same
Solidity in our view means \dot{y} quality by \dot{w} no two
parts of matter can exist in \dot{y} same place, or \dot{y}
power by \dot{w} M^r excludes all other matter from
its place. we acquire \dot{y} notion by our sense of touch,
and had we never touched a body, our sense of seeing
never could have given us any Idea of it.

An artful painting, the picture in a camera
obscura, & the figures in \dot{y} air formed by a concave
mirror have every other appearance, & might
have suggested to us all its other properties.

But had a person passed his life only among such
appearances he could have acquired no Idea of solidity.
And yet some philosophers have attempted to
deduce this property from extension because no
two points of space can occupy \dot{y} same place this is
very true, but arises from \dot{y} immobility of the
parts of space, & no one can doubt of the spectrum
formed by mirrors existing in \dot{y} same place.

It is \dot{y} particular view of solidity \dot{w} is called The im-
penetrability of m^r & \dot{w} is asserted to be an universal
property of m^r . If a body were not impenetrable
any pressure would be sufficient to destroy it.

\dot{y} matter \dot{w} composes the globe, & \dot{w} is all
biased by a tendency to \dot{y} centre, presses on \dot{y} inferior
parts \dot{w} very great force, & yet an annihilation of them
does not follow we justly conclude \dot{y} all \dot{y} m^r of the Earth
has this property. As \dot{y} accurate examination of \dot{y}
celestial phaenomena shews us \dot{y} there is the same
tendency of \dot{y} m^r \dot{w} composes the body of the sun & the
planets to their respective centre we justly conclude that
all m^r has \dot{y} property. Altho \dot{y} resistance \dot{w}
we feel opposed to our attempts to compress m^r is \dot{y}
original of our notions of \dot{y} property it does not inform
us of its nature; it is true \dot{y} if we only suppose \dot{y} \dot{y}
parts cannot change their respective situation it
will give us a distinct notion of impenetrability.

Now \dot{y} only way in \dot{w} we can suppose \dot{y} is to conceive
 \dot{y} \dot{y} parts of M^r are in contact; for sensible measures
of m^r from \dot{w} by analogy we from our conceptions of its
atoms, are not possessed of such a power, but may be
made to change their situation by very moderate
forces, except when in contact. This is therefore \dot{y} most
natural way of accounting for \dot{y} impenetrability of
body, & by \dot{w} we explain it.

Let us examine it.

There is hardly a body \dot{w} does not admit a passage

thro' it to some other body, & they sit
together - Thus water penetrates into marble
Gold is penetrated by mercury tho' both are extre-
mely compact - one fluid penetrates another - light and
fire penetrate all bodies, what is more to the present
purpose - In many mixtures of metals in many
mixtures of fluids, in many cases of solution, if mix-
ture occupies less room than than the sum of its
ingredients - can the particles of any body come nearer
than contact? this is impossible?

All this is allowed; but it is answered by saying
that if parts of m^r touch only in a few points, &
leave great interstices between them. The other body
intersects these, and is lodged there - Thus a box filled with
musquet balls will yet receive a considerable quantity of
small shot, to which we may afterwards add a consi-
derable quantity of sand, and after all a quantity of water -

But this does not explain some of the difficulties. There
occur in Chemistry some facts where a fluid will occupy
a smaller space after another body has been added. This
happens in the case of water and some salts, & in the case
of air & its principle of inflammability. If a diamond is hard-
est of all bodies be struck on a piece of marble it will rebound
this it cannot do without compression, nor does it avail to
say if its parts are not brought nearer by its compression,

is bent, for it remains to shew how they can be
brought without being compressed in some parts. Take if
some diamond, measure accurately its bulk, expose it to
a great degree of cold, & it will be found to have contracted
all its dimensions. This it could never do had the parts
been in contact - I must here observe if its resistance
is we feel in our attempts to compress body does not allow
us to conclude if its parts of m^r touch each other; for there
are many instances of this resistance where we know if
the bodies do not touch. If its impossibility of reducing
water to smaller dimensions leads us to think that if
particles do not touch the impossibility of doing if
beyond a certain degree in vapour & air should lead us
to form if same conclusion. Where we hang up a chain,
we think its links touch each other, for they support each
other but electrical experiments shew if they do not touch
till forced together by a great weight, & it is not demon-
strated that they do touch. Optical experiments shew
if two object glasses of long telescopes do not touch
even when pressed to a very considerable weight, &
even then we can shew if even tho' if appearances
of if want of contact cease it does not follow if they
touch.

From all these we may conclude it does not
arise from contact of parts nor authorises us to
say it is parts of body touch.

I thought it is best & shortest way of saving you
a great deal of trouble of reading & attempts of phi-
losophers to account for the external form, & many of
its sensible qualities of bodies from its form of their parts.
Very hard bodies are by them supposed to consist of
parts which adhere by many touching parts. Brittle bodies
have few points of cohesion. Water is fluid & almost with-
out cohesion, because its parts are globular. But plaster
of Paris when very hot exhibits every appearance of
fluidity, tho' its parts are rough and kept asunder
by the acid vapour which comes thro' them in steam. It
is evident that all forms are indifferent if its parts do
not touch, nay if it can be demonstrated that they cannot
touch.

These considerations will also save you the trouble
of reading & attempts of philosophers to explain the constancy
of natural productions from the perfect hardness of
elementary particles. Now there is no doubt but
perfect hardness is a necessary quality of an elementary
because they consist of calx & phlogiston.

It does not change either its external form or
chemical properties. There is no more reason for sup-
posing it necessary that if calx and phlogiston is composed
an atom should touch, than that if atoms should
touch in order to form a sensible mass.

Accordingly it has been the opinion of some eminent
philosophers that if parts of matter are not in contact.
Sir Isaac Newton expresses his opinion in this
manner - Suppose that God had endowed mathematical
point with insuperable powers of repulsion extending to
minute distance only, what Idea could we acquire from
it but that of solidity - Sir J. N. and M^r Lock conversing
on this subject shewed that from it we could form a notion
of creation. It was only necessary that God should render im-
penetrable what was before penetrable. For that purpose it
was only necessary to ordain that no other portions of space
should not penetrate a particular portion of space & then
that portion becomes impenetrable. Many look on creation
as an absurdity, but in this view it does not appear so.
The difficulty of any operation proceeding from certain
obstacles, and mere nothing opposing no obstacle, it appears
that creation brings neither absurdity nor difficulty. This
consequence is of principle advantage if we can reap from
the opinion of these philosophers.

The celebrated Leibnitz also supposed impenetrability. He supposed if universal composed of elements he called monads, kept asunder by spheres of activity, if were not unmountable, and they might therefore be forced to unite or penetrate. The person who has taken if most accurate view of if subject is father Boscovich. who has built a system of Nat. Philosophy on if principle, if he endeavours to support by many arguments drawn from abstract considerations, and he shews several ways in if most perfect impenetrability may be produced without if smallest continuous extension of if elementary parts of m^r. But at if same time he observes if this absolute impenetrability does not probably obtain in nature but that every mass of m^r is compressible, if is will allow if freest passage to every other mass of m^r without disturbance of if parts of either; precisely in if same manner as we see if rays of light pass freely thro' transparent bodies without if smallest disturbance of their parts or of its own motion. He supposes matter to consist of nothing but mathematical points, kept at their proper distance by certain powers of attraction & repulsion, so if each point is in if limit between an attracting & repulsive power. It will not approach if other on account of if repulsive power. Now says he let another particle of such matter approach slowly to a body. as soon as it comes into the sphere of repulsion of the part of that body it velocity will be diminished while

it pushes if particles a little from each other; if motion is stoped & it does not penetrate. If it moves with a greater velocity it will be able to approach nearer & separate if parts farther but yet stops. If it moves still swifter, it may be just able to pass thro', but at last extremely slow. During all the time of its passage it is employed in separating if particles of the body. Suppose it moves still swifter, it passes thro' more quickly, & as its repulsive power act on if particles of if body for a shorter time, it disturbs them less from their place. This disturbance will be lessened by every augmentation of its velocity, & when if is enormous, it will pass thro' without any sensible effect on if body. This he illustrates by if well known fact of a musket ball discharged on an open door, if it penetrates without making it move on its hinges; whereas were its velocity much less it would shatter it exceedingly, & give it a sensible motion. This is a fact well known to if sailors who observe if a great shot almost spent does much more danger to if ship than when it comes to its greatest force. It is in this manner if light & heat penetrate bodies without any sensible disturbance of their parts, on account of their almost immense velocity. From such considerations it must follow if by sufficiently augmenting if velocity of any body we shall not only enable it to overcome the impenetrability of bodies but even to pass thro' them without disturbing their parts or itself suffering any disturbance.

You must remark \bar{y} all \bar{y} may happen with
penetration of \bar{y} material particles; namely \bar{y} particles
are only points, it is impossible \bar{y} \bar{y} point of different colour
can touch, and therefore \bar{y} compenetration is consistent both to
 \bar{y} apparent but false impenetrability is we usually ascribe to bodies
& also to its real impenetrability.

If \bar{y} particles of m^r are not mathematical points,
but physical points of their minuteness, endued wth such
powers of attraction & repulsion as I have been mentioning,
& w^{ch} I will afterwards explain at great length, all the
sensible properties of m^r will follow, without any contact
w^{ch} I apprehend you will soon be convinced is a thing w^{ch}
does not exist in nature. You see then \bar{y} by all that I have
said on this quality considered as a distinguishing property of m^r .
I have adhered to \bar{y} vulgar notion of it meaning by it
nothing more than this fact \bar{y} m^r opposes w^{ch} attempt
to penetration, & \bar{y} what manifests \bar{y} truth of \bar{y} opinion
is matter. I may only observe that it is a name for
a primary quality of m^r , and not a name for a sen-
sation of \bar{y} mind. When I speak of \bar{y} smell of a
rose, I commonly do no more then name my sen-
sation but when I speak of the solidity of a diamond,
I speak of a real quantity of \bar{y} body, w^{ch} existed tho'
it had never been handled, altho' the smell of a rose
has no existence but in \bar{y} mind of him who smells.
The sensation by w^{ch} we perceive \bar{y} solidity of m^r

ately has got no name because we seldom
know it, but pass immediately from it to the thing
perceived, the solidity of \bar{y} body we can however fix our
attention on it if we please, & find it totally unlike \bar{y}
thing of w^{ch} it conveys \bar{y} notion to our minds. we probably
did attend to it at first, while we were infants, & employ-
ed in making ourselves acquainted to our means of
information, tho' we have afterwards in most cases totally
overlooked it; just as a person in learning a language
first attends to \bar{y} sounds only, & afterwards neglecting
them entirely, turning his whole attention to the thought
the thing signified by the sound.

I shall only farther observe that this
quality of m^r is one of w^{ch} we have \bar{y} distinctive Idea.
It is not like colour, sound, smell, of w^{ch} we know nothing, &
about w^{ch} philosophers have formed many hypothesis.
If \bar{y} parts of m^r exclude other m^r from their place, it is
solid when I pretend to go a little farther than \bar{y} vulgar, &
shew that we conclude very hastily when we infer from \bar{y}
solidity of m^r \bar{y} its parts are in contact, or when we deduce
solidity from \bar{y} contact, and when I ascribe \bar{y} quality to
certain powers of attraction & repulsion, I do not assign a
cause, because in saying \bar{y} a repulsive power keeps
asunder the particles of m^r I only say \bar{y} they keep asunder.

In calling \dot{y} discontinuity of the parts of m of power, it is only from the analogy of language. \dot{y} is every thing a power by which animal power is opposed, & we may therefore call \dot{y} quality a power meaning by \dot{y} term only \dot{y} m is solid. I formerly observed \dot{y} is expression of a coincidence in matter of fact was called a natural law. I therefore assume as \dot{y} first law of nature.

That all matter occupies its own place to the exclusion of all other m . and \dot{y} no body can penetrate another, so as to exist along with it in \dot{y} same place. Since we find \dot{y} in a known space we can put very different quantities of m . & yet sensibly fill it, as appears by \dot{y} box filled wth musquet Balls, and afterwards admitting \dot{y} addition of small shot; since we find \dot{y} a cubic foot contains a quantity of water when cold w^{ch} will do more than fill it when hot, and since we find \dot{y} it is highly probable that \dot{y} particles of m are not in contact, & \dot{y} we are uncertain at what distance they are placed from each other, it follows \dot{y} in a given bulk there may be more or fewer particles of m . The mass, or number of material particles of \dot{y} a body consists depends on \dot{y} space it occupies, or its bulk, & what is called its density.

any given bulk there is \dot{y} greater number of particles, or \dot{y} greater quantity of m ; & nearer they are to each other. It is \dot{y} vicinity w^{ch} we express by \dot{y} term density. we say \dot{y} a body is denser than another body if its particles are nearer to each other, & rarer, if they are farther removed. Now we do not know their distance, we cannot say that one is denser than another except by finding \dot{y} a given bulk has more of some other sensible quality w^{ch} is competent to all matter. Thus for instance, supposing \dot{y} all particles are equally heavy, we say \dot{y} m has twice the density of \dot{y} a given bulk has twice the weight - and thus density w^{ch} really means \dot{y} closeness of \dot{y} material particles, becomes like velocity a term of relation, expressing the proportion w^{ch} \dot{y} number of particles of m in \dot{y} same bulk of another body.

In thus representing \dot{y} density as expressing \dot{y} number of particles contained in a determined bulk w^{ch} we call unity, it is plain \dot{y} in order to have \dot{y} mass or total number of particles of m in any body we must multiply the density by the bulk.

We shall for η future use M to express η ,
 m , and Δ to express the density, & B to express the
bulk from η arises η following formula

$$M = B \Delta, \quad \Delta = \frac{M}{B}, \quad B = \frac{M}{\Delta}$$

Always remembering η by density we do not mean
the real number of particles, but only proportion
of η number in a given bulk of one body to the number
of parts contained in an equal bulk of the other body

II. Second of η perseverance of m . vid. book from page 13
to page 15

Corollary 1. A body η is perceived at rest, has always been
at rest, unless affected by some external cause. For if not, it must
have come into its present place either from one side or
from another; but there is no cause to determine from η side it
should come - it must therefore have come from neither, or
must have been at rest.

2. If a body has acquired motion from any external cause,
it cannot accelerate or retard this motion - for it cannot
determine itself either to motion or rest.

3. Whenever a body at rest is brought into motion we ascribe this to
some external cause.

Proposition 2.

In order to preserve a body in motion, η instantaneous action

ing cause is sufficient. This is very different from
opinion entertained by some philosophers, who imagine
 η continual action of η cause is necessary, & thence find themselves
obliged to ascribe all motions to η perpetual agency of intelligent
beings. Other philosophers, who see one reason for having recourse
to the immediate action of deity or intelligent beings, ascribe
 η continuation of motion to η continual agency of some power
inherent in m , called into exertion η moment that η body is
put in motion, & keeping exact proportion to whatever is the
original cause of η motion, so that when η original cause is
double, & η motion is double, the continual exertion of this in-
herent power shall also be double. But these opinions are in
themselves extremely obscure & do not tend to remove what
is here thought to be a deficiency, for it will appear that
 η instantaneous action of η moving cause is not sufficient
to make the body continue in motion, neither η continued action of η .
Motion is conceived by all as η effect of a cause, and there would
be no room to infer a cause if there was no effect, that is if
a body were not moving. Now, motion is a change of place,
or is η successive existence of a body in different parts of
space. Apply this to η present case. The action of η moving
cause produces motion in η body, that is causes η body to
be in a different place now from what it was in before
now η motion is successive & continual. If therefore η
action of η cause in any one instant does not produce a

continual change of place, it does not produce
motion, & is not a moving cause. Continual
motion therefore is implied in the very notion of a
moving cause, & is a consequence of its instantaneous action
otherwise its instantaneous exertion of its moving cause
would be intirely without effect. And if its instanta-
neous exertion is without effect, its continual exertion
must also be without effect. for since in no instant
of its continuation it produces any effect, it is impossi-
ble that it can produce any by its continuance. It there-
fore motion exists by instantaneous exertion of its moving
cause is sufficient for producing a continuation of motion.
There is not therefore any necessity of supposing in a mo-
ving body any power by which it continues in motion, or for
having recourse to the continual action of external causes;
& its exertions of such power or such external causes
would be ineffectual.

Proposition 3

See book from page 17 to 19. The demonstration of
these fundamental propositions proceeds on the truth
of its axiom that

Let any force Q act in the direction AQ and let the free motion which it would produce be AQ . Drawing the necessary lines as before, AB will represent the rotatory effort, and AX the pressure on the Centre of Motion.

AB is to AB in the compound ratio of AB to AF , AF to AQ , and AQ to AB . That is

$$\begin{aligned} & \text{of } PD : PA \\ & \text{of } FA : QA \\ & \text{and of } PA : PC \end{aligned}$$

That is of $PD \times FA \times PA : PA \times QA \times PC$ or $PD \times FA : QA \times PC$.

What is, in general, the rotatory effects of powers applied in this Manner are as the products of the forces by the perpendicular distances of their directions from their centre of rotation.

On the other hand supposing that their perpendicular distances are the same, but that the distance of the body is changed, as to PC (Fig 28) then calling the effort in the former case E , and the effort in the present case e , and the perpendicular distance P ,

$$E : e = PA : P$$

$$e : E = P : PC$$

$$\text{Therefore } e : E = PA : PC$$

That is when the perpendicular distances are the same the Efforts are inversely proportional to the distance of the points of Application. Call this distance D ; then the effort in all cases, may be expressed by this formula $\frac{P \times P}{D}$, which will represent the Quantity of Motion communicated to A in the direction

direction of the tangent by any power communicating force F

Further as this quantity of Motion communicated to A is proportional to the product of its Matter M by its velocity V And as this velocity will consequently be reciprocally proportional to the quantity of Matter, when the motion F , or $\frac{F \times P}{D}$ is given it follows that the velocity which any force can give communicated in this Matter will be directly proportional to the Quantity of Motion F , and inversely as the Quantity of Matter; therefore we shall have this proportion $V \propto \frac{F}{M}$ or $\frac{F \times P}{M \times D}$.

Lastly, as this Motion of Rotation is an Angular Motion, it is not measured by its Angular velocity, but the Angular velocity of any Body describing a Circle is as the Arch directly and the distance from the Centre inversely, or as its real velocity directly and the distance inversely, that is in the present case as $\frac{F \times P}{M \times D}$ directly and D inversely, or as $\frac{F \times P}{M \times D^2}$. we shall find this formula of the most extensive use in practical mechanics.

Let us now consider the resistance which the Body A makes to this rotatory Motion of F . It must be considered as precisely equal and contrary to the motion, that is it must be proportional to $M \times D^2$; And since $F \times P$ is constant it must be proportional to $M \times D^2$.

is another way of conceiving where resistance may in every case whatever be considered as a power opposed, contrary and precisely equal to the motion impressed. For a double body offers a double resistance to any given velocity, and any body opposes a double resistance to a double velocity. A body A at rest when hit by an equal body B moving with a velocity 1 takes away from its velocity a velocity $\frac{1}{2}$. If B moves with a velocity 2 it takes away a velocity 1 double the former.

Now the dimensions of B 's velocity are the measures of the resistance of A 's inertia. Therefore the effect of A 's resistance in this case of rotatory motion is proportional to $M \times D$ (for $F \times P$ is constant and does not change the proportion) and its effect is the same as if A were not resist but were but were held back by a power proportional to $M \times D$. Now the effect of this power $M \times D$ is proportional to the perpendicular distance of its direction from the Centre of rotation. But since the motion of A is along the Arch of the circle, always perpendicular to PA its perpendicular distance from P is D . Therefore the effect of this power in opposing the Motion of rotation is $M \times D \times D$ or $M \times D^2$.

While therefore the effect of the power F in producing a rotation is expressed by $F \times P$, the effect of A 's inertia in opposing it will be expressed by $M \times D^2$. This truth is also of the most extensive use in practical Mechanics.

Suppose now that the force F instead of acting immediately on the body, acts on it by the intervention of immaterial and inflexible line AF , the effect must be the same. Suppose that PD also be line an inflexible line, connected with PA and AF . The effect must remain the same. Suppose that the parts AD of the inflexible line AF is annihilated, but that the line DPA is inflexible. The effect must still remain the same. Lastly suppose that the force F , instead of acting on D by the intervention of the line DF , acts immediately upon it. The effect must still remain the same. It follows therefore that the Motion produced in the body A is the same whether the power F acts immediately on, or acts immediately on D .

The resistance opposed by the inertia of A , may be considered as a power equal and contrary to the motion produced in A . Let then be applied to any other part of the line PA for instance O a body whose resistance in opposing the motion of rotation shall be precisely equal to that of A . we do not yet know what proportion this body must bear to A , in respect of its distance from P , it is enough that it opposes at A a resistance equal to that of the body A applied at A . Let now the power act again at A it must have the same effect as before in producing a rotation round P , since the resistance is the same. But it will have the same effect if applied at B . From this arises a most useful truth, that the effort of a force in producing a rotation round a fixed point, in opposition to a resistance exerted to prevent this Motion, is always

proportional to the perpendicular distance of its direction from the Centre of rotation, and that it is indifferent to what part of the assemblage of inflexible lines connecting the power, resistance, & Centre it is applied. Or in other words, the effort which any force exerts in turning an assemblage of inflexible and immaterial lines round a fixed point, is proportional to the perpendicular distance of its direction from that point.

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^{Fig 29}
If from any point P , be drawn PD, PE, PF perpendicular to the sides and diagonal of any Parallelogram $ABRC$, then the rectangle of the diagonal and its perpendicular shall be equal to the sum or to the difference of the Rectangles of each side and its respective perpendicular according as the point P lies without or within the Angle A thro which the diagonal passes. That is $AR \times PE = AC \times PF \pm AB \times PD$.

Case 1st. Let P be without the Angle BAC . Draw PA, PB, PC, PR , to the Angles of the parallelogram, and let RB cut PF in G . Then the Triangle APR , or $\frac{AR \times PE}{2}$ to the triangle $ABR + PBR + PBA = \frac{AC \times GP + BR}{2}$ or $\frac{AC \times EG + AB \times PD}{2} = \frac{AC \times PF + AB \times PD}{2}$ double the whole, and then $AR \times PE = AC \times PF + AB \times PD$.

Case 2nd. It is demonstrated in the very same Manner. Let there be now / Fig 30 / two forces HK, IL , having the proportion and directions of their lines, act on the Assemblage of lines HP lying in the same plane

plane and produce a rotation round P in oppo-
sition to some resistance opposed, for instance, at
point O of the inflexible line PO connected with them.

Produce KH, IL till they meet in A, make AB, AC
 $= KH, IL$, and completing the parallelogram
draw the diagonal AR. Draw PD, PE, PF \perp to AB,
AR, AC. Then $AR \times PE = AC \times PF + AB \times PD$ or AR
 $\times PE = IL \times PF + HK \times PD$.

Let more PD, PE, PF be-
come inflexible lines connected with PH, PI. Then
the efforts of HK, IL, acting at D, F are equal to the
effort of AR acting at E. Now the efforts are the
same should they act at H and I and AR at C.

Hence we deduce the general Theorem that the
effort of any two forces acting in any direc-
tions lying in the plane of rotation, is equal
to the effort of their resulting force which is always
equal to the sum or difference of their particular
efforts according as the centre of rotation lies
without or within the angle formed by their
directions, or according as they tend to turn the
lines the same or contrary ways.

The same may be demonstrated of the force resul-
ting from the combination of any number of
forces acting in the plane of rotation.
And this effort is equal to the sum of all
the efforts tending one way — the sum of
all the efforts tending the contrary way. The point
O thro' which the direction of the resulting force
passes is called the centre of Effort of these
forces.

There is one case to which the foregoing reasoning
does not immediately seem to apply. Namely when
the forces are parallel to each other, in which case
they do not meet in any Point such as A. But
we can easily find the centre of effort of such forces.
Since these directions are parallel then, the line
which cuts any one of their directions HK, IL
at right angles, cuts also the others IL & TS at
right angles. In order to take in all the varieties,
we shall suppose that the force TS acts in a
contrary direction to the forces HK, IL.

Let us suppose that O is the point thro' which
their resulting force OR passes: this must be
parallel to the others, since there is no reason
that it should incline more to the one than to the
other. This may be rigidly demonstrated. Let
there be two forces AB, AC their directions always
passing thro' the points F, H, and AB keeping its
direction and their resultant AR (Fig 32) with the
centre A, and distance AF describe the Circle FDE,
cutting AR in D, and AH in E. Draw DG, DH perpen-
dicular to AF, AK, and EI perpendicular to AF
then the lines DG, DH, EI are proportional to
AB, AC, AC, AR, because the forces represented by
the sides and the diagonal of a Parallelogram are
always proportional each to the sine of the angle
contained by the other two. Then as the point A
moves off, and the angle A diminishes, the line EI
continually approaches to an equality.

and coincides with the arch E, D, F and is ultimately equal and coincident with it when AE is parallel. In like manner DG, DH ultimately coincide with the arches D, F, D, E , and are equal to them, and these arches are ultimately coincident and equal to the straight line FL perpendicular to AF , and since each of the lines AE, AD, AE is perpendicular to the arch on which it stands, the three lines AE, AD, AF are ultimately perpendicular to FL . AD is therefore ultimately parallel to AE and AF , and ET is ultimately equal to DG & DH and AB, AC, AR are ultimately proportional to MF, ML, LE . Hence it follows

1. The resultant of two parallel forces is parallel to them.

2. It is equal to their sum or difference according as they act one or different ways.

3. If a line be drawn across their direction, each is proportional to the part of it intercepted by the direction of the other two.

In the line HT take any point P . Then from the property of the resulting force we now demonstrate.

$$HK \times HP + IL + IP - TS \times TP = OR + OP.$$

Take any other point p then

$$HK \times Hp + IL + Ip - TS \times Tp = OR + Op.$$

Therefore $HK \times Pp + IL \times Pp - TS \times Pp = OR + Pp.$

and consequently $HK \times IL - TS = OR$

And $HK \times HP + IL + IP - TS \times TP = OP$

$$HK \times IL - TS.$$

since we see 1.st That the resultant of any number of parallel forces is equal to the pre valent sum of the particular forces, and acts in that direction

2. The distance from any point is had by dividing the prevailing effort by the pre valent sum.

These things may be demonstrated without the Lemma shew. Let their directions meet in A as before,

and suppose a body in A (Fig 33) connected with the inflexible line PH, PI by means of the invariable angle IPA , and such as to oppose by its inertia a resistance equal to the resistance with

to conceived to have been opposed; to the AB, AC, HK, IL , and completing the parallelogram AR will be the resisting force. Draw BD, CE, RF

perpendicular to the tangent AF, AB, Ch, Ri perpendicular to AP . Then AF will be the effort of the resulting force in producing a rotation

round P and Ri will be the pressure which it occasions on P . Now AF evidently is equal to $AE \pm AD, \& Ai = Ag \pm Ah.$

Cor. 1. If the diagonal, or the direction of the resulting force passes thro' the centre of rotation, there will be no motion, for then the perpendicular from the Centre of rotation vanishes, which

Makes one of the factors of the general effort.

Or in the other way of conceiving it, AF vanishes which represents the motion of the resisting

body

Body applied at A, and is equivalent to the axis
against the resisting power.

In this case as no rotation occurs, the powers
are in equilibrio with each other which they
would not be without this connection established
between them by the inflexible lines. There is
therefore an equilibrium whenever when the
direction of the resulting force passes thro' the
Centre of rotation, and in no other. In this case
too the Centre sustains the whole pressure of the
resulting force, and it is easy to see that this
pressure is the same as if the particular forces
had been immediately applied to it in their own
directions. This appears by making the parallelo-
gram $PQER$ equal, similar, and similarly
situated as $ABRC$. I may observe in general,
even when a rotatory motion happens, this is
the case if we consider (which we ought) the
resistance opposed to the rotation as one of
the powers applied. For make $PM=AI$,
and PN equal and parallel to AI , and PQ equal &
parallel to AR and join QM, MN , it is evident
that it constitutes a parallelogram having
lines equal and parallel to AF & AR for its sides,
and AI for its resultant, the pressure on P .
This observation is of the greatest use in the construc-
tion of machines. What has been hitherto said is
true only of forces which act in the plane of rotation,
but

But suppose they act in parallel planes, and
that the rotation is not performed round a
point P , but round an axis passing thro' P perpen-
dicular to those planes. The result and the de-
monstration will be precisely the same, and the
only variety which this Circumstance introduces
is the equilibrium of the ends of this axis, since we
see that, by considering the resistance as one of
the principle powers applied, we may consider
all the forces as immediately applied to this axis, and
then making any plane pass thro' it, resolving
each force into two, one of which shall be parallel
and one perpendicular to this plane, and all of
them perpendicular to the axis, we can easily
find the two points of the axis thro' which the
resultants of each set shall pass, and these determine
the equilibrium of the axis.

Should the powers not act even in parallel planes,
we can resolve each into three, one of which shall
be in the parallel planes of rotation, and
then we shall find the state of motion, or of equi-
librium as before. but I must leave this to your
private consideration, contenting myself
with having pointed out to the method of
investigation.

with

With this I conclude what I had to say concerning the motions of single bodies considered as all collected in their Centre, and now proceed to consider the laws which regulate the general motions of Systems.

Of the motions of a System.

In order to discourse on the motion of a system of bodies, it is necessary to attend both to its general progressive motion, and to the relative motion of its parts. In order to investigate the general motion, it is evident we must select some point of the system, by whose motion we may esteem the motion of the whole. Now in every system of bodies a point may be found such that supposing the whole matter of the system were collected there, and moving with its motion, the whole motion counted in any direction will be the same with the real whole motion counted in the same direction.

Such a point may therefore properly be called the centre of the system, and it is worth while to examine into the method of ascertaining in what part of the system it may be found.

It is easy to see from what was formerly said of the centre of a single body, that the centre of a system is to be found in the very same manner. It will be a point such that if a plane be made to pass thro' it, and from every particle of matter in the system be drawn a perpendicular to this plane, then the sum of all the perpendiculars on one side will be equal to the sum of all those on the other side.

But a circumstance occurs which renders the investigation of the Centre of a system much easier. It was shown that if from every particle of matter in any body were drawn a perpendicular to any plane whatever, the sum of all these perpendiculars was equal to the distance of the centre of that body from the plane multiplied by the quantity of matter. In order therefore to find the centre of a system, we may consider each body as contracted to its own centre, and then the centre of the system will be such a point, that if any plane pass thro' it, and the matter of each body be multiplied by its distance from that plane, the sum of the

Products on one side will be equal to the
of the products on the other.

Let A, B, D / Fig 35 / be any three bodies any
how placed and let C be the centre of this
system. It has this property similar to
the centre of a single body, that if any plane
E.F be drawn, the sum of the products of each
body multiplied by its distance from
this plane is equal to the sum of the bodies
multiplied by the distance of the centre
from the plane.

For draw thro C a plane parallel to E.F viz KL

$$\text{Then } A \times AI = B \times BK + D \times DL$$

$$\text{now } A \times AE = A \times CH + A \times AI$$

$$\text{and } B \times BF = B \times CH - B \times BK$$

$$\text{and } D \times DG = D \times CH - D \times DL$$

$$\text{Therefore } A \times AE + B \times BF + D \times DG = A \times B \times D \times CH$$

$$+ A \times AI - B \times BK + D \times DL.$$

$$\text{but } A \times AI - B \times BK - D \times DL = 0$$

$$\text{Therefore } A \times AE + B \times BF + D \times DG = A \times B \times D \times CH$$

hence CH may easily be found it being = to

$$\frac{A \times AE + B \times BF + D \times DG}{A \times B \times D}$$

Therefore to find the distance of the cen-
tre of the system from any plane, multi-
ply each body by its distance from that plane,
and divide the sum of the products by the sum
of the bodies - the Quotient is the distance
sought. Hence we have an easy Method
of finding the centre. Find its distance
TI from any given plane, viz ABDE
the centre must be in some point of a plane
OPQR, parallel to the given plane, and hav-
ing this distance from it; then find its distance
TO from another given plane ABFG. It is generally
convenient to make this plane perpendicular
to the former of them the centre must be some-
where in the plane IKML parallel to this second
given plane and having the distance now
now found from it. It must therefore be in the
common intersection TW of the two planes hav-
ing the distance now found from given planes
Find now its distance LD from any third
plane BDHF. It is generally convenient
to make this plane perpendicular to the two
former. It will therefore be somewhere in a
plane YZ ab parallel to this third plane,
and having the third found distance from

from it. It will therefore be in the point of section C of all the three planes, and thus its place is determined. We must now attend to some geometrical properties of this point.

1. The centre of a system of two bodies lies in the line joining their centres, and their distances from it are reciprocally proportional to their quantity of Matter, or to the bodies.

Prop^y 37. Join the centres of two bodies A & B and divide the line in C so that $A : B = BC : AC$. Then draw thro' C any line; then $BE' : AD = BC : AC = A : B$ and therefore $A \times AD = B \times BE'$ which is the property of the Centre.

2. Let now a third body B (Fig 38) then the centre of the system of three bodies is in the line joining the third body with the centre of the other two. Let C be the centre, and joining it with D produce it towards E . Then since D is a plane passing thro' the centre, if we draw AF , BE' perpendicular to it, we must have $A \times AF = B \times BE'$ & $A : B = BE' : AF$ therefore $A : B = BC : AC$, &c. with the centre of A & B .

In the next place C & D is divided by the centre of the system C that $A \times B : D = DC : CC$. For Fig 39 draw thro' C the plane EF perpendicular to DC , and draw the perpendiculars AE , BE . Then since C is the centre of A , B it follows that $A \times AE + B \times BE = A \times B \times CC$.

But since C is the centre of A , B , D , $A \times AE + B \times BE + D \times DC = A \times B \times CC + D \times DC$ that is $A \times B \times CC = D \times DC$ and $A \times B : D = DC : CC$.

3. In like manner it may be shewn that if a fourth body E is added the common centre of this system is in the line joining E with the centre of the other three and so situated that $A \times B \times D : E = EC : CC$. And so on of any number of bodies. (Fig 40)

5

II, of the perseverance of matter
or of property by it m^o always continues in its
state in it is whether of rest or of motion.

Every body at rest will continue in its state unless it
is disturbed from it some external cause.

For simplicity's sake, and in order to rid ourselves
from every foreign circumstance, I shall suppose it
there exists nothing else in nature but its body of it
I am speaking; & of consequence it is without weight,
it is a quality bearing an unvariable reference to its Earth
an external body. I farther suppose its body at perfect
liberty, exposed to no friction or resistance all its flow
from external things.

Now if it is asserted that a body in circumstances can deter-
mine itself to motion, it must move either to one side or
another. But there is no cause it will determine it to
one side rather than another; & as no determined event
can be conceived to happen without a cause of determination
it follows that it will neither move to one side nor to another
i. e. it will remain at rest.

Prop. III

A body put in motion by any cause whatever, will continue to move uniformly in a straight line, unless it is acted on by some other external cause.

For after the first instant of action of its moving cause exists no longer & yet its motion subsists but a body in motion cannot accelerate or retard its own motion, for it must either accelerate or retard it, and as there is not supposed an intervention of any cause to determine it to acceleration, more than to retardation or the contrary, its motion during this time must have been uniform.

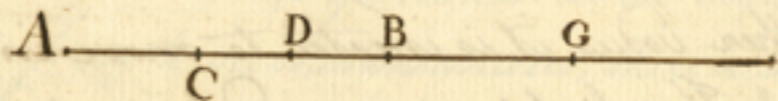
In like manner as no cause is supposed to intervene to determine it to deviation to one side more than to another, it must have moved in a straight line.

Since then it can move for any time, however small, independent of its moving cause, it must move during its time uniformly in a right line.

But a body which has moved for one moment uniformly in a straight line, must continue to do so, if nothing disturbs it.

For supposed a body setting out from **A**, & capable of moving uniformly thro' **AB** in any time however small — Take any

points **C, D** between **A** & **B** — the body when at **D** is, in every, in the same condition as when at **C** — But at **C** it was in a condition to move uniformly to **B** in a certain time — at **D** therefore it is in a condition to move uniformly thro' an equal space in an equal time; that is it is in a condition of itself of moving uniformly thro' **DG = CB**, & so on & thus a body in motion tends to move uniformly in a straight line till disturbed by some external cause.



Hitherto we have been considering a body as brought into motion by the instantaneous action of one single power. But by much of greatest number of phenomena of nature shew us bodies moving in consequence of the combined action of several powers - a ball is projected in the air is at once acted upon by the impulse it acquires from my hand, & by its own weight - the one would carry it forward in the direction in which it was projected, & the other would make it fall to the ground. When a body receives at once a stroke from two other bodies it is incited to move in the direction of both strokes - If a lighter is dragged by two horses on different sides of a river it tends to both.

But in all these cases, the body cannot follow the action of either power, but must move in some third direction - whatever this direction is, & to whatever velocity the body moves, its direction & velocity, in short its motion must be considered as resulting from & equal to the motions which are impressed on it at the same instant.

The motion which the body takes is considered as compounded of two or more motions which it would take by the action of each power singly, & the joint action is called the composition of motion.

Fig. 2 The general question is this. Suppose if a body A is acted on at once by two forces P, Q , one of which P would carry it uniformly from A to B in a certain time, & the other Q would carry it from A uniformly to C in the same time, what will be the path & velocity of the body? This question is of the utmost consequence in Nat. philosophy, & the greatest part of what will follow will be only making applications of it. It is evident the path must be a straight line, & the motion must be uniform. For as we suppose the joint action of two forces to be instantaneous, there occurs nothing afterwards to induce any change.

Fig. 2 Let AG be the unknown path & at any point G of this path let two other forces act on it - one that would carry it along GE equal & parallel to AC , but in the opposite direction, in the same time the body would have moved along AC by the force Q alone & another that would carry it along GF equal parallel & contrary to AB . It is plain that the body must rest in the point G . For had those new forces acted on it at A , at the same time to Q & P , it must then have rested, as the equal and opposite forces must have destroyed each other. But the body at G is in precisely the same situation as at A , for it has the same velocity & direction as at A .

& is as if actuated by ² two forces \vec{w} acted on it at A ; that is by two forces equal & contrary to GE & GF . Whatever therefore is the joint effect of the two forces GE & GF is the joint effect of AC & AB must be equal & contrary. Let us therefore substitute for GE & GF two forces whose joint effect we perfectly know.

For this purpose let us suppose the A adheres to a plane void of weight & inertia, & therefore incapable of inducing any change on the motion of A . When A has got to G , suppose that this plane is carried along the grooves $MILK$ parallel to AC , in a contrary direction & with a velocity equal to the velocity along AC , in this case it is evident that the point G of the plane will describe GE equal & parallel to AC , & therefore the body will have impressed on it a force equal & contrary to the motion along AC . Suppose further that the grooves MI, LK are carried along ML parallel to AB with a velocity & direction equal & contrary to the motion along AB in consequence of this second motion the body has impressed on it a motion = & contrary to the motion along AB . Now it is unnecessary to demonstrate that the joint effect of these two motions must be a motion along GH , the diagonal of the parallelogram $FGEH$. Since therefore the joint effect of the forces P, Q must be equal

& contrary to this, it follows that if a body actuated by them at A must describe a line AD equal & parallel to HG , but in a contrary direction. This must evidently be the diagonal of the parallelogram $ABDC$.

I therefore assume it as a 3^d law of motion that
"If a body is acted on at once by two forces each of which would make it describe the sides of a parallelogram in a uniform motion in a given time, it will by their joint action describe the diagonal of the parallelogram uniformly in the same time"

we see then that it is indifferent whether we conceive the body A as sollicitated by two forces P, Q , or by one in the direction AD & fit for carrying it along AD in the time that P, Q would have carried it along AB, AC .
The two or more forces which combine in producing a motion along some indeterminate direction are called the simple & constituent forces, & the motion which results from their joint action is called the compound, or resulting motion. And as the motion might have been produced by a force in its own direction, & having it for its measure, it is called the equivalent, or resulting force. The conceiving two motions or forces

combined is called *composition of motion*, & *conceiving a motion or force as resulting from a combination of two or more motions, or forces, is called resolution of motion & forces*. As it is of so much consequence in the science of motion, & the greatest part of what is to follow is a continual application of it, it is necessary to give it a particular attention — Therefore observe —

1st That *constituent & resulting motion & forces acting on a material point are all in one plane.* For *its sides & its diagonal of any parallelogram are all in one plane.*

2^d The *constituent & resulting motions are = to its sides of a triangle* if they are parallel or any how = by inclined to its directions —

For AB, AC, AD (fig. 3) are = to AB, AC, CD , because $CD = AD$. Now if all its sides of its triangle abc are parallel to its respective sides of its triangle ABC , its triangle are similar & its sides proportional. The same may be said of its triangle $\alpha B\alpha$ whose sides $\alpha B, B\alpha, \alpha C$ are respectively perpendicular to AB, BC, CA — & the same may be said if they are any how equally inclined —

3^d The *constituent & resulting motions are each equal to the sine of its angle made by its directions of its other two*. For (fig. 3) its sides of any triangle are = to its sines of their opposite angles, therefore $AB:AC::\text{Sine } \angle C:\text{sine } \angle B$; & $AB:BC::\text{Sine } \angle C:\text{sine } \angle A$ —

4th If two forces conspire in its same direction they will produce a motion equal to that produced by a force = to their sum — or if two motions conspire in one direction its resulting motion will be = to its sum —

For let BAC , its inclination of its constituent forces AB, AC (fig. 4) continually diminish & at last vanish, in its case these forces or motions have but one direction AC —

It is evident its AB becomes AE, BD becomes EF , & its resulting force AD becomes $AF = AB + BD, = AB + AC$ —

Hence it follows its joint action of two forces conspiring in one direction is = to its sum of their separate actions — or more properly its velocity communicated by its joint actions of two conspiring forces is = to its sum of its velocities communicated by their separate actions.

It appears almost needless to have made this observation — but it is of the greatest importance to our just conception

of its actions of forces. It has been generally assumed on
its credit of its maxim its effects are proportional to its
causes, a maxim is, to say its best of it, is a mere tauto-
logy, in its sense in which it is used in Nat. philosophy: for we
know nothing of its proportions of its causes but its propor-
tions of its effects. It is a maxim too which has led some
of its most eminent philosophers into great errors, & has
occasioned great disputes in philosophy which never can be
can be terminated till the maxim is given up.

S. J. N. has introduced it into its science of motion under
its name of a law, in these terms — The change of motion
is \propto to its force impressed

But in all its use which he makes of it he never takes any
other view of its degree of its force impressed than its change
which it produces. Leibnitz & others, reasoning from its same
maxim, asserted, that its forces are not \propto to its velocities
which they communicate, but to its squares of these velocities;
for there are many cases, in which we think ourselves certain
of its proportion of its acting forces, & we find the velocities
which these communicate in its subduplicate ratio of its
forces from which these philosophers have been led to one

of these two conclusions, either that its velocities are not the
proper measures of its effects, or that there exists in a
moving body a force differing from it which is impressed on it.
But its present determination was founded on no hypo-
thesis concerning its nature of forces, or any previous
knowledge of their proportions. All that we know of a
force is that it produces motion, & it is conceived as a force
only in so far as it produces motion; and from this
we are led clearly to the conclusion that its forces are
really \propto to its velocities which they communicate, provided
only if we call its a double or triple force which is $=$ to its
sum of 2 or 3 = forces: Now we have no other notion of
its proportion of any quantity whatever.

5. If two forces act in opposite directions its velocity,
which they communicate is equal to that which would be
communicated by a force $=$ to their difference — or if
two motions are in opposite directions its resulting
motion is equal to its difference.

For let **BAC**, (fig. 5) its inclination of its constituent
motions **AB**, **AC**, continually increase till it becomes 100° ,
then **AB** becomes **AE**, **CD** becomes **CF**, **DB** becomes

FE, & if resulting motions AD becomes AF = AC -
DC = AC - AB

Hence it follows if $AB = AC$ the diagonal AD vanishes, that is rest ensues from if joint action of if constituents AB, AC. This is called if equilibrium or ballance of motions & moving powers, & it is if foundation of a principal branch of Dynam. & if partical mechanics. The construction of machines the construction of arches, if stability of ships & many others of if most useful application in Nat. Philosophy to if arts of life depending on if principle of if ballance of powers - all may be reduced to one generical proposition -

6. Three forces or motions which have if directions & proportions of if sides & diagonal of parallelogram will be in Equilibrio, provided if if force is has if proportion of if diagonal acts in an opposite direction to if diagonal. For AE (fig. 6) is if motion resulting from if motions AB, AC; but if AD is equal & opposite to AE, it will (by if last observation) ballance it - It will therefore ballance if two motions AB, AC.

7. Let it now be req. to find the motion or force resulting from the joint actions of if motions or forces AB, AC, AD, AE & (fig. 7)

Complete if parallelogram BACE, & it appears if AF is if motion or force resulting from if joint action of AB, AC. Complete if parallelogram ADGE, & then AG is if resulting. force of AF, AD, or of AB, AC, AD. Complete if parallelogram AE, HG & AH is if resultant of AE, AG, or of AB, AC, AD, AE

8. In these compositions of motions it is by no means necessary if if constituent motions AB, AC, AD, AE, by in one plane; for however they be, still if part^m ABFC will be in one plane, & AF will be if resultant of AB, AC. In like manner if part^m ADGE will be in one plane, whatever is if position of AD, & if part^m AEHG will be in one plane whatever is if position of AE. On if other hand if motion AH may be resolved into any two AE, AG, provided only that they are if sides of a par^m of it is if diagonal; and AG may on the same conditions be resolved into any two AF, AD, & AF may be resolved into any two AB & AC - & thus for AH

may be substituted AB, AC, AD, AE , which will produce the very same effect.

9. The most useful of all \dot{y} resolutions of forces is \dot{y} following — Let AB (fig. 8) be any force or motion produced by it; Let CD, EF be two lines cutting each other in P at right angles, & let GPH pass thro' P at right angles to \dot{y} plane in \dot{y} CD, EF are — Let it now be req^d to resolve \dot{y} motion AB into three, w^h shall be respectively parallel to PG, PC, PF . Thro' A draw AI, AL, AM parallel to PG, PC, PF . From B draw BK parallel to IA & meeting \dot{y} plane in w AL, AM by in K , draw KA & draw BI, KM, KL parallel to KA, AL, AM . Then it is plain \dot{y} for AB may be substituted AI, AK , & for AK may be substituted AL, AM — therefore for AB may be substituted AI, AL, AM — To give an instance of \dot{y} application; suppose a ship sailing at right angles to \dot{y} wind; in this situation her stay sails have a threefold operation. They partly tend to drive her forward, in \dot{y} direction PC partly to drive her to leeward, in \dot{y} direction PF , a very detrimental effect, & partly to lift her up in \dot{y} direction PE , & thus diminish \dot{y} resistance of her prow. In order to reap all \dot{y} advantages of this noble machine it is necessary that \dot{y} sail should be so disposed, \dot{y} it effect in \dot{y} direction

PF should be as little as possible & \dot{y} effect in \dot{y} direction PC should be as great as possible —

In like manner \dot{y} action of \dot{y} sun on \dot{y} moon (fig. 2) in \dot{y} direction MS tends partly to pull her down to \dot{y} Ecliptic in \dot{y} direction MP , partly to pull her away from \dot{y} earth in \dot{y} direction MC , & partly to pull her back in her orbit, in \dot{y} direction MD — The motion of \dot{y} moon cannot be ascertained till it is determined what proportion of \dot{y} whole disturbing force of \dot{y} sun is employ'd in producing each of these effects.

10. Any number of motions or forces AB, AC, AD, AE , will be ballanced (fig. 7) by a force AK equal & opposite to their resulting motion or force AH — This needs no demonstration for it follows from observation 6 —

Such then are \dot{y} general Laws of Motion, \dot{y} principle from w^h we are afterwards to reason synthetically in introducing \dot{y} subordinate laws of motion, in investigating \dot{y} powers & in explaining the phaenomena of Nature.

Of the free motions of a single body

And here at first setting out I must observe, if a smallest conceivable body may be considered as a system, and during its general motion of its whole, its different parts may have their own motion. It is necessary to abstract entirely from these varieties and to fix on some one point of its body by whose motion we may esteem its general motion of its body.

In every body a point may be found, such as if a plane be made to pass thro' it, and if from every particle of matter there be drawn a line perpendicular to this plane, the sum of all its perpendiculars on one side shall be \propto to its sum of all those on its other side.

If another such plane be found then the point will be in the intersection of these two planes.

If a third such plane be drawn then the point will be in that point of this third plane where the common intersection of its other two planes passes thro' it.

This point I call the centre of the body, & it has the following geometrical properties in order it convenient for our purpose.

1. If from every particle of matter in the body there be drawn a perpendicular to any plane tw in or tw out its body, its sum of them all is equal to its distance of this point multiplied into the quantity of matter. — For (fig. 10) draw thro' C the plane DE parallel to it B & from P , a particle on its same side of DE without AB draw its perpendicular FPE , & from H , lying on the other side, draw the perpendicular HKI . Then its sum of all the lines such as FE = the sum of all its lines such as PE + its sum of all its lines such as FP ; and the sum of all the lines such as KI = to its sum of all such a HI - its sum of all such as HK - but by the property of its centre its sum of all the FP = to its sum of all its HK .

Therefore $\int FE + \int KI = \int PE + \int HI$ - but $\int FE + \int KI = CE \times \int F + \int K = CE \times \int P + \int H = CE \times M$ expressing the quantity of matter or number of particles P & H .

2. If its body moves in a straight line the sum of all its motions of each part equal the motion of $C \times M$. This will be true even tho' its body should during its motion turn round C ; for its sum of all its retrogradations of its

particles which go backward may be shown to be equal to \dot{x} sum of all \dot{x} accelerations of \dot{x} particles is come forward, and it will afterwards be demonstrated that the body moving freely can turn round no other point but C — DE

Therefore the motion of \dot{x} whole body will always be equal to \dot{x} motion of C x M —
 Thus then the distance of the body from any plane, & its motion may be conveniently and accurately esteemed by \dot{x} distance & motion of C, & therefore \dot{x} whole body must be supposed to be reduced to \dot{x} point C —

During \dot{x} time represented by LM; but it terminated in P, and KP is \dot{x} space really described —

Lemma Fig. 11

If AB, BC, CD, &c are taken to represent any successive portions of time, and BE, CF, DG &c are the spaces run over uniformly from the beginning of \dot{x} times, the points E, F, G, will be in a straight line. For the spaces are \dot{x} to \dot{x} measures of \dot{x} times, & therefore AB:AC = BE:DF therefore, by a known theorem A, E, F are in one straight line.

The same may be shewed of G and of any other point. BE, HF, KG are the spaces described during \dot{x} portions of time represented by AB, BC, CD —

If BC = CD, then HF = KG. The velocity v is any part HF is described = $\frac{HF}{BC}$ — This then will serve as a very convenient characteristic of uniform motion, that if any line such as AD is taken for \dot{x} measure of \dot{x} time, & ordinates are drawn from its different points equal to \dot{x} spaces described in the portions of time counted from \dot{x} beginning, the extremities of all these ordinates will be found in a straight line —

On the other hand if, after having taken such a line to be \dot{x} measure of \dot{x} time, the extremities of \dot{x} ordinates are not found to lie in a straight line the motion is not uniform. Thus (fig. 11. no. 2) if the points A, E, F, G, O, P, Q are

found not to be in a straight line, the motion during the time represented by AN is not uniform. If between A & F they are found to be in a straight line, then the motion has been uniform during the time represented by AC ; produce AF till it cuts DG in g . Had the extremity of DG been in g , then the motion during the time represented by AD would have been uniform, & hg would have been the space described in g portion of time represented by CD . But since g extremity of g ordinate is in G , hG is g space described during the part of time represented by CD . Now hG exceeds hg by gG ; the motion therefore during g time represented by CD has been swifter than before, and g motion has been accelerated at g instant of time represented by C .

If FG, O , be in a straight line, then the motion during g time represented by CL has been uniform, tho' swifter than before.

Did g ordinate MP terminate in P in GO produced, then the motion during the time represented by DM would have been uniform, & Kp would have been the space described during that time and is less than Kp by Pp . The motion therefore has been retarded at g instant of time represented by L .

Such motions come next to be considered.

Of Varied Motions

Motions are susceptible of two variations Velocity & Direction.

We shall first consider g variations in velocity, or the acceleration and retardation of rectilinear motions, these are susceptible of infinite varieties & may all be investigated in the same way by the proportions of g abscissa and ordinate of such figures as I have now exhibited. In order to shorten language, I shall for the future drop g consideration of AB, BC, CD & as measures or representations of the times, & I shall simply say the times AB, BC, CD & c.

Had g motion during the time BCD been uniform the body would have described the space hg in the time CD , but since it has really described g space hG, gG is g effect of g acceleration. But when I say that gG is the effect of g acceleration, it does not of itself convey a precise notion of this acceleration, unless I consider also g time in which hG has been described. — The velocities taken in both circumstances of space & time, and is therefore the proper measure of g acceleration — the velocity in which hG is described is $\frac{hG}{CD}$ & $\frac{gG}{CD}$ is the proper measure of the acceleration happening at g instant C . In like manner $\frac{PP}{LM}$ is the proper measure of g acceleration at the instant L . In the present example it is evident that g acceleration

has happened by pts. at \dot{t} instants C & L. & \dot{t} during each of the intervals AC, CL, LN, if body has moved uniformly, for \dot{t} velocity during \dot{t} time AC is constant, since $\frac{BE}{AB} = \frac{CF}{AC}$ in like manner because $\frac{hG}{CD} = \frac{LQ}{CL}$, \dot{t} velocity during CL has been constant.

But suppose that no part of the line which passes thro' \dot{t} extremities of the ordinates is straight (fig 12)

Suppose for instance that the motion is continually accelerated - and let BC, CD be two equal portions of time and BE, CF, DG be \dot{t} spaces described during the times AB, AC, AD - draw EH, FI parallel to AD; then HF, IG are \dot{t} spaces described during the equal times BC, CD; and therefore since the motion is accelerated IG must be greater than HF - The points E, F, G therefore will lie in a curve convex towards AD.

In this case it is plain that we have no constant measure for \dot{t} acceleration - If we take $\frac{IG}{CD}$ for the velocity at the instant C it will be too great and $\frac{HF}{BC}$ will be too small - Draw MFL touching \dot{t} curve in F, and cutting BE, DG in M, L - let now \dot{t} points B, D continually & equally approach \dot{t} point C & ultimately coincide w' it, then the points M, L will continually approach, and ultimately

coincide w' G, E - MN is always equal to FI & NF is always equal to IL, therefore $\frac{NF}{MN}$ is always equal to $\frac{IL}{FI}$; Hence it follows \dot{t} either of \dot{t} quantities, suppose $\frac{IL}{FI}$, will be \dot{t} proper measure of \dot{t} velocity at \dot{t} instant C; but IL in \dot{t} case is as \dot{t} fluxion of \dot{t} space CF, & FI as \dot{t} fluxion of \dot{t} time AC. Hence we have this formula for expressing \dot{t} velocity in any accelerated motion whatever $V = \frac{\dot{S}}{\dot{t}}$

With respect to \dot{t} acceleration itself, it consists in the augmentation made in the velocity. But an acceleration that is continual & perhaps unequal will produce different augmentations of velocities in different times. In order therefore to be free of such irregularities it is necessary to suppose \dot{t} time of the acceleration extremely small, and even to consider it as evanescent - In \dot{t} case \dot{t} acceleration will be directly as \dot{t} increment of \dot{t} velocity & inversely as \dot{t} increment of \dot{t} time; or $\dot{t} \propto \frac{\dot{V}}{\dot{t}}$ - Did \dot{t} body at the instant C go on to move during \dot{t} small moment CD w' \dot{t} velocity w' it had at \dot{t} moment C then IG, or \dot{S} , would be the space w' it would describe in \dot{t} moment CD, or \dot{t} - But as \dot{t} motion is accelerated, IG is \dot{t} space described, & $LG = \dot{S}$ is \dot{t} acceleration - but LG alone does not convey a precise notion of \dot{t} acceleration at \dot{t} instant C - The velocity is \dot{t} proper measure w' is $\frac{LG}{CD}$; were FgG a straight line, then $\frac{LG}{CD}$ would be a constant quantity.

whatever were of motion CD - But FL is not to FI as LG to Lg , & therefore $\frac{LG}{FL}$ is not a constant quantity & a proper measure of its acceleration. But it was formerly demonstrated that its ultimate \div of LG to Lg was equal to the of FI^2 to $Fü^2$, & therefore $\frac{LG}{FI^2} = \frac{Lg}{Fü^2}$. This quantity therefore $\frac{LG}{FI^2}$ is a proper measure of its acceleration at the instant C ; but $FI^2 = CD^2 = t^2$, & LG is evidently the increment of IL , it is its increment of CF - LG therefore is \dot{S} - Hence it follows that $\frac{\dot{S}}{t^2}$ is its proper measure of the acceleration.

But we shall get simpler & more convenient measures - For its purpose recollect its $v = \frac{\dot{S}}{t}$ (C) therefore $\dot{S} = vt$ - therefore $\dot{S} = vt + t \dot{v}$ -

But let us suppose t constant; it is convenient on many accounts, for in comparing variable quantities it is always necessary to have one constant quantity.

2. we are now prepared for investigating its nature of accelerating forces. I have frequently taken occasion to observe its we have no knowledge of its pro. of nature but by their effects - The powers by its its variety of motion its we observe is produced are known to us in no other way, may we know nothing of them but their effects, or its change

of motion its they produce, we judge of its existence of an accelerating pt^l only by observing the acceleration, & of its degree only by its quantity of that acceleration - when we say its its pt^l of gravity acts on a stone we only mean its its stone falls, or tends to fall - when we say its gravity is greater at its poles than at the equator we only mean its a stone falls faster or farther in its same time.

From its it happens its its formula $\frac{\dot{S}}{t^2}$, its I have just now given for its measure of its acceleration, is assumed by all its mechanical writers as its measure of its force its is conceived to produce its acceleration - In its they have proceeded on its truth of maxim its causes are proportioned to their effects - but its is a beginning the question, if we know nothing of the nature of causes or of its principle its connects them to its effects; & till we know this we are in no wise intitled to affirm its causes are proportional to their effects.

You will soon see cases where its maxim has led into every great errors & into disputes its will never terminate till the maxim is given up as a mere tautology - When Sr J. Newton established his second law of motions

* "that the changes of motion were proportioned to \dot{v} causes it produced them he did not foresee \dot{v} obuse it was to be made of it; & in all \dot{v} uses it he has made of it he never means any thing by his cause but \dot{v} effect itself.

I intend therefore to keep altogether clear of \dot{v} danger, & for \dot{v} future; when I use \dot{v} word force as signifying a physical quantity, & assign it certain proportions according to \dot{v} variable circumstance of \dot{v} case, I desire to be understood as meaning nothing more than to express by one word not \dot{v} cause of a change of motion, but \dot{v} change of motion itself. It is in \dot{v} sense that I now say \dot{v} \dot{v} accelerating force is proportional to $\frac{\dot{v}}{t}$ or to the increment of velocity communicated in a moment of time.

This shall always be \dot{v} meaning of the formula $F = \frac{\dot{v}}{t}$; Taking \dot{v} view of accelerating forces we have an easy method for comparing \dot{v} to each other. Let F & ϕ be two such forces, \dot{v} & \dot{u} \dot{v} velocities it they communicate in \dot{v} infinitely small times t , T ; then $F : \phi = \frac{\dot{v}}{t} : \frac{\dot{u}}{T}$.

It is convenient to choose some determinated time, such as a second; therefore t being = T $F : \phi = \dot{v} : \dot{u}$.

That w^t
& m

Mechanical properties

of the centre of a system of bodies

Let any two bodies move uniformly in any velocities and directions AD, BE , in any planes, then if following will be the motion of the centre C of this system

1. It will either rest or move uniformly in a right line CO given in position & magnitude

Complete the parallelograms $ADFC, BDGC$, each of which will be plane figures. Join FG, DE & draw COQ thro' their intersection. CO shall be the path of the centre of the system & its motion will be uniform

Let if bodies come to H and K in the same time. Draw HI, KL \parallel to AB - Join IL , cutting CO in S (it will cut it, for the triangle FCO is in one plane) draw SH, SK

Since if bodies move uniformly, $AH:AD = BK:BE$ & $CI:CF = CL:CG$, and therefore IL is \parallel to FG .

Because BF is \parallel to GE , $DF:GE = DO:OE$ but $DF:GE = AC:CB$ - Therefore $DO:OE = AC:CB$, & O is the centre of the system when the bodies are at D & E ; and $FO:OG = AC:CB = HI:LK = IS:SL$. But if angles HIS, KLS are equal, on account of the parallels HI, KL . Therefore the triangles HIS, KLS are similar, and if angles ISH, LSK are equal,

and therefore HSK is a straight line, & since HS:SK
 $KL = AC:CB$, S is the centre of \dot{r} system when \dot{r}
 are at H and K. The same may be shown when the bodies
 are at \dot{r} same time in any other parts of \dot{r} lines AD, BE.

Therefore the centre moves in a straight line. Further, CS:
 $CO = CI:CF = AH:AB$. Therefore the centre moves uniformly

2. Their quantity of motion estimated in the direction perpendicular
 to the path of their centre is nothing. For A has moved in this
 direction thro' a space = FN and B thro' a space = GQ. then
 $FN:GQ = FO:OC = B:A$. Therefore $A \times FN = B \times GQ$, & their
 difference is equal to nothing.

3. Their quantity of motion estimated in the direction of the
 path of \dot{r} path of \dot{r} centre, is equal to the quantity of motion which
 would have been produced had both bodies been collect. at \dot{r} centre,
 & moved to its velocity.

Thro' O the final place of \dot{r} centre draw a plane $HI \perp CO$

Draw AF, DH, BG, EI parallel to CO, and draw
 DK, KL perpendicular to AF, & BG.

Then A has advanced in \dot{r} direction of CO
 thro' a space equal to AK and its quantity
 of motion, estimated in \dot{r} direction
 of CO is $A \times AK$, that of B is $B \times BL$. Now by \dot{r} properties of \dot{r}
 centre, $A \times AF + B \times BG = \overline{A+B} \times CO$. and $A \times DH$, or $A \times KF =$

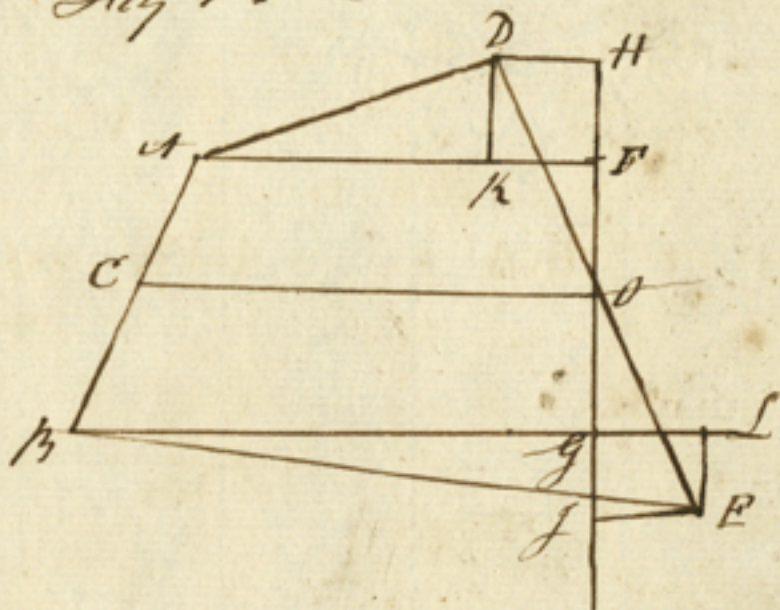
that is
 I , or $GL = O$; therefore $A \times AF = A \times FK + B \times BG + B \times GL =$
 $A + B \times CO$ or $A \times AK + B \times BL = \overline{A+B} \times CO$.

Cor. 4. The velocity of \dot{r} centre will be had by dividing \dot{r}
 prevailing quantity of motion of the bodies, counted in the direction
 of \dot{r} centres motion, by \dot{r} sum of \dot{r} bodies.

4. If any new force is impressed on one of the bodies, the motion
 will be affected in the same manner as if it had been impressed
 in a parallel direction on all the matter collected at the centre,
 & moving to its velocity.

Fig. 42. Let AD, BE be the paths of the bodies, & CO that of \dot{r}
 centre. And when \dot{r} bodies are at A & B, let an external force
 act on B such that by its action alone B would move thro' B β
 uniformly in the time in w^h it would move uniformly along BE.
 Complete the parallelogram B β EE, then BE will be the path
 of B. Draw DE & make DE:DO = DE:DO, and draw CO, CO will
 be the path of the centre of the system. For O is its place at the
 end of the motion, and it has been already demonstrated that its path
 will be a straight line. Complete the parallelogram COox,
 then $Cx:B\beta = Oo:E\epsilon = DO:DE = B:\overline{A+B}$ therefore $\overline{A+B} \times$
 $Cx = B \times B\beta$. Therefore had the force $B \times B\beta$ been applied a C to
 $\overline{A+B}$ at rest it would have caused this mass to move along
 CX, and if applied to them in motion with the direction and

Fig 13



velocity CO , it would carry them thro CO . Wherefore
position is manifest.

Cor^o. If any motions be impressed on both bodies whether at
once or separately, & motion of the centre will be affected in the
same way as if the forces had been immediately impressed
on the whole matter collected there and moving in its
velocity & Direction.

All that has now been demonstrated of the centre of
a system of two bodies, may be extended to any number
of bodies. For we may consider a system of three bodies as a
system consisting of the 3^d body and the other two united in
their centre. In like manner when a 4th body is added &
Hence we deduce the following general Theorems.

1. If any number of bodies are moving uniformly in straight
lines their common centre either rests or moves uniformly.
2. The quantity of motion of the whole system, estimated in a
direction perpendicular to the path of the centre, is nothing.
3. The quantity of motion of the whole system, estimated in
the direction of the path of the centre, is equal to the sum or
difference of the particle motions, estimated in the same
direction according as they all tend one way or contrary ways.

The velocity of the centre is to be found by dividing their quantity
of motion by the matter of the system.

4. The quantity of motion of the system, counted in the direc-
tion of the path of the centre, is the same to the quantity of
motion of the whole matter moving to the velocity of its centre.
5. If any forces whatever act on the bodies of the system, its
motion of its system, estimated by the motion of the centre
is the same as it would have been had its same forces been
applied to its whole matter of its system & moving in its
velocity & direction.

Having shewn the properties of the centre of a system
of bodies detached from each other, let us attend to the
consequences of their mutual actions. It is this is properly
constitutes a system, and these mutual actions may be very
different according to the principle it connects the different
parts of the system. The bodies in compose a system may
be connected by tendencies to approach or avoid one another,
commonly called powers of attraction & repulsion, such
as obtain among magnetical & electrical bodies.

In this case each body is at full liberty to follow the
combined motion of all the forces it act on it, & its
path is not restricted. Or a system may be connected
by means of inflexible & inextensible lines.

In this case the bodies are not at liberty to follow pulses of the impressed forces, but any force impressed on any body is immediately diffused over the whole system. Or lastly the connection of the system may be composed of both of these. We shall consider these in their order.

And first in respect to the motion of a system where each body moves freely.

Suppose that for a moment all the mutual forces should cease; then each body would move on uniformly in the direction in which it was moving the moment before, and the centre will either rest or move uniformly in some straight line. Restore now the system action between any two

of the bodies & suppose first that it acts only on one of them. The centre will be affected in the same manner as if all the matter of the system was collected there, and this action was exerted on it. But it was formerly shewn that in the mutual actions of bodies there is an equal & contrary reaction. The body which exerts the accelerating force is equally affected in the opposite direction. Thus occasions an equal & opposite motion of the centre. In consequence therefore of these equal & opposite motions the centre will rest. It is not affected therefore by the mutual action of any two bodies.

The same may be applied to all the bodies and it therefore follows that the motion of the centre of any system is not affected by the mutual equal actions of its bodies on each other. It therefore either rests or moves uniformly in a straight line.

We demonstrate in the same manner that the quantity of motion of such a system, estimated in the direction of its path, is the same as if the whole system were collected there, and moving with its velocity & direction.

And we demonstrated that if any external force is impressed on the system, its motion of the centre is affected in the same way as if the force had been impressed on the whole matter collected there & moving in its velocity and direction.

Cor^y. The centre of the whole universe either rests or moves uniformly in a straight line.

By this means we are enabled to transfer what was formerly demonstrated concerning bodies revolving round fixed centres by means of forces directed to them to bodies revolving round each other by means of mutual attractions & repulsions. It must that such motions, as we then considered hardly obtain in nature. Attractions & repulsions, are not exerted by points but by bodies. Further it must

be observed that in mutual attractions & repulsions
ting body, which we conceive as producing the revolution
The other. cannot remain at rest, and therefore the centre of
ces will no longer be a fixed point, if no force besides their mutual
attraction and repulsion acts on them they will approach or
recede from each other, with accelerated motion, and velocities re-
ciprocally proportional to their quantity of matter. Should
any oblique act on one or both, they will acquire a motion round
their centre, while it will either rest or move uniformly in a
straight line.

How these motions may be adjusted comes next to
be considered.

I. Two bodies mutually attracting each other describe round
each other figures similar to those described round their
common centre.

1. They will describe round γ centre similar figures for
the line joining the bodies always passes thro γ centre, and it
is not moved by the mutual action of the bodies. Also their
distances from this centre are always in a given ratio, namely
 γ inverse ratio of the bodies.

6. That when there are many ways in w^{ch} an event may happen,
& no cause determining to any, the event itself is impossible.
But this must not be looked as if cause of if perseverance
of a body in a state of motion and rest till disturbed by some
external cause the cause must be if nature of m^r; & if is
only if means of demonstration.

It is not however allowed by all if such demonstrations are
irrefragable,
which are of if kind & to have had no determining cause to
some modes of their existance. Thus in if immensity of space
and time there appears no cause w^{ch} should determine Duty
this place or this time or w^{ch} if world should exist, & therefore if
existence of the universe is an impossibility.

Altho' if must be answered by attending more accurately to the
nature of intelligent beings, I will not trouble you w^{ch} if discussion
w^{ch} would be long & tedious. I shall rather proceed to lay before you
if experimental proofs of if proposition. But here I must be
allowed to mean only experimental proofs if these properties
of m^r w^{ch} respect to motion, and w^{ch} flow from if nature of
motion, do really obtain in the world. Because there w^{ch} is
absurdity in supposing, if whatever are if natural conse-
quences of m^r left to itself, yet the author of nature must
have imprinted, & perhaps has implanted very different
laws on it. The whole universe, from if moment of its
creation, may be in a continual progressive motion of if

we must forever remain ignorant, & there may be implan-
ted in m^r a power of beginning, regulating, & terminating
its own motions, as well as there is a method a power
of beginning regulating, & terminating its own exertion.
And accordingly if doctrine has been advanced by one of
most penetrating Metaphysicians it has ever appeared
& has so little absurdity in it as to have obtained its suffrage
of some of the greatest geniuses of Europe.

Such of You as have been much engaged in philosophical
inquiries must have heard of the system of M^r Leibnitz,
who made his universe to consist of certain elementary
beings which he called monads, & which among other extraordi-
nary properties, were endowed with powers of changing their
situation, & were continually exerting these powers. I am
apt to think that his disputes with Sir J. N. about the
invention of fluxions led him not only to cavil at his phi-
losophy in general, but in particular at this principle,
which makes its basis of it. The system is now entirely forgot-
ten, but if supports it were given to it were ingenious,
& all concurred in shewing it there was no absurdity in
supposing m^r endowed with powers of changing its situation.
If a vessel of water is suddenly drawn along
its floor it leaves part of its water behind. If it is suddenly
stopped, part of the water flies ~~outward~~ out forward.

If persons are in a coach or boat, & if coach or boat begins sud-
denly to move, they fall backwards. If it suddenly stops
they fall forward. If a peice of money is laid on a smooth
card, & if card struck smartly horizontally, the card flies away
& the money remains, shewing at the same time the tendency
of m^r to remain in motion, & to remain at rest. If I
throw any thing straight out of a coach window while the
coach is going briskly but smoothly along, it falls oppo-
site to the window, altho the coach has advanced consi-
derable while the body was falling. You have seen a man
dismount & remount while his horse was at gallop you
have seen him spring from one horse to another in the same
situation. You have seen a man on the slack wire, or on
horseback playing with oranges thrown up in the air, which
fell always into his hand as if he had been at rest. None of
these things could happen if the man in mounting or dis-
mounting, & the oranges while in the air did not preserve in their
motion which they had while connected to his hand or his horse.

A stone from the hand or from a sling, an arrow from a bow
all move on by persevering in their state of motion in so they were
left by his hand, his sling or the bow. On the other hand, were
it not for the tendency to remain at rest, a bullet would not make
a hole in a wall, but carry the wall along with it.

houses would not fall by earthquake but would move along
to γ ground.

Thus the fact seems supported by a large induction
of experiments.

But the bulk of mankind think γ rest is γ natural
state of body - that nothing is necessary for a bodies conti-
-nuing at rest but its continuing to exist. Whereas when a
body is put in motion an acting cause is necessary for γ con-
-tinuance of γ effect, it is always new, always beginning. And
this common opinion is also founded on a very large induction
of experiments. We see, say they hardly any motion in
nature it does not greatly tend to rest a billiard ball rests at
last however strong γ stroke - a pendulum ceases to move if
not continually acted on by γ machinery of γ clock - In
short, no machine no body does continue its motion it out
a renewal of γ moving cause.

But I must note γ all γ observations to γ purpose are
particular, whereas γ observations for ascertaining general
laws should be general in all these particular observations
we can see causes for γ deviation from the assertions.

The retardations it we observe are in all probability owing to γ
action of known obstacles roughness of rubbing paths, resist-
-tance of bodies & γ like. For in such cases as we can make ex-
-periments, & remove these obstacles the motion continues so
much longer. The smoother & more slippery γ surface on it

a body moves, it longer does it motion continue - the rarer the
medium in it it body moves, it more lasting it motion as
in mercury, water, air, void. There is seen a top it will
continue to turn in vacuo for a whole day. Since then we
see it a diminution of the obstacles is invariably attended
to a diminution of it retardation, there is reason to ascribe
it retardation to these obstacles, and to conclude it were
they intirely removed, it motion would continue forever.
And it is proved by it planetary motions, it being perform-
-ed in a space void of resistance, have continued unchang^{ed}
ever since Astronomers could accurately observe their
motions.

In all it cases too of deviation, from uniformity,
either of velocity or direction, we find a constant concomitan^{ce}
between it deviation & something external, & it it different
degrees of these deviations, where they can be examined with
accuracy, bear an invariable relation to it changes of
these external relations, & when these external relations
are removed, it deviations are removed at it same time.
Thus it curvilinear path of all bodies projected in it air
in a direction not perpendicular to it horizon bear an
invariable relation to it position of it path to respect to it
lines in it gravity acts, & by varying these positions we
vary at pleasure it curvilinear path of it projected body.
The same may be said of every other case.

So far then common experience seems perfectly consistent to *it* abstract truth. But *it* experiment alluded to are inaccurate, & only concur in leading us to conclude from observation that *m*^r has a tendency to continue in its present state; but by no means give us any defined notion of *it* tendency.

May if we come to consider *it* motion with accuracy we shall find *it* such observations are totally unfit for deciding *it* present question, & *it* we are not authorised to make the deduction from them. You will find it afterwards demonstrated that if once a particle of *m*^r in *it* universe has begun to move it is impossible *it* any one particle should be at rest, unless it should happen to be in one only point of the universe. Rest therefore, so far from being a familiar situation to *m*^r is one in *it* we have never found it, & therefore were we to judge from these observations only, we would affirm *it* a general law of motion *it* rest was incompatible to *m*^r.

We shall find too *it* every case *it* we look on as an instance of rest is really an instance of most rapid motion: *it* many cases *it* we take to be instance of passing from motion to rest, are in fact instance of a very great increment of motion, & *it* we are totally insensible of changes of motion inconceivably greater than any *it* we can observe.

But *it* reasoning would have been quite *it* same had *it* experiment been performed at any other time, & with any other direction only not so simple, on account of *it* obliquity of *it* relative & absolute motions.

Now when *it* experiment is tried, *it* attention to every circumstance which can introduce any variety in *it* result is invariably found to be *it* *it* quantity of motion lost by *it* overtaking body is precisely equal to the quantity of motion gained by *it* overtaken body, & *it* conclusion therefore drawn from it is *it* tendency of *m*^r to continue at rest is precisely equal to *it* tendency to continue in motion.

We may therefore assume it as *it* first law of motion that every body perseveres in its state of rest or of uniform rectilinear motion, unless disturbed by *it* action of some external cause.

And we may hence see *it* *it* laws of motion established by *it* author of nature are *it* same to what result from *it* nature of motion abstractly considered. Arguments to *it* same purpose have been drawn from *it* exact agreement between *it* phenomena of nature, & what would be *it* phenomena ~~if nature, & what would be *it* phenomena~~ supposing *it* law took place.

house would not fall by earthquake but would move along

But *ij* argument is not conclusive. for whatever have been *ij* laws of nature w respect to motion *ij* agreement of *ij* phaenomena to *ij* calculation would have been *ij* same.

Because assuming any law at pleasure, & meaning every disturbing cause by *ij* deviation is it produces from *ij* law *ij* result of our conclusions would have been *ij* same, tho' *ij* quantities introduced into our calculation would have been different. And this leads me to consider *ij* causes of the change of motion.

III. of the powers

and activity of matter.

From *ij* first law of motion it follows *ij* motion. and every change of motion is to be ascribed to a cause. To *ij* cause we give *ij* name of power. The term is analogical, borrowed from animal life. we call *ij* quality of *ij* mind by which it is fitted for producing motion in *ij* body, or at least we call that faculty of an animal by which it is fitted for putting *m^r* in motion by the name of power.

By analogy we call *ij* quality of any cause by which we conceive it as fitted for producing its effect by *ij* same name & we make a similar application of *ij* term to *ij* quality in *m^r* by which it is fitted for producing motion in *ij* *m^r*. Animal power exerted is termed force & by analogy when *ij* powers of *m^r* are exerted in producing motion, they are termed force, in relation to *ij* body ~~is it is~~ moved, but powers in relation to *ij*

body which is the cause of *ij* motion. We find *ij* same exertions of animal powers necessary for stopping motion or *ij* tendencies to motion. By analogy we call *ij* quality of *m^r* by which it diminishes or stops motion, or *ij* tendencies to motion Force.

When speaking in reference to *ij* body stopped or having its motion diminished, but powers when speaking in reference to the body employed to stop or diminish them.

As *ij* tendency in *ij* *m^r* manifests to continue in motion is seen in *ij* change which it produces in other *m^r*, we consider it as a power, & as employed in producing this change, we consider it as exerting force. as *ij* is conceived as residing in *ij* moving

body, it is called *ij* *Vis insita*, *ij* inherent force of *ij* body. But a want of accuracy in our view of *ij* matter has led some eminent writers into mistakes. Matter is

carelessly said to have a power by which it preserves itself in motion.

But since all agree in giving *ij* name of power to what produces a change of motion, it is inaccurate & illogical to give *ij* same name to what is not employed in producing a change. The

continuation of motion therefore indicates no power, but is *ij* very effect of *ij* moving cause, by which I mean what originally produced *ij* motion. In strict language therefore *Vis insita*

means *ij* power by which *m^r* in motion puts another body in motion.

As *ij* tendency of *m^r* to remain at rest is manifested in

producing change in \dot{y} moving body, it is also carelessly
termed a power of continuing at rest, & is termed vis
inertiae. In strict language it means only \dot{y} power
by \dot{y} a body at rest diminishes \dot{y} motion of another body.
It is also called vis resistentiae, because its effect &
operation are analogous to what we call resistance in animal
life. Some use \dot{y} term inertia (first introduced by Kepler)
to express both \dot{y} power manifested by \dot{y} moving & \dot{y} mani-
fested by \dot{y} resisting body to maintain their present condition
and Sir J. N. applied it indiscriminately to \dot{y} tendency
 \dot{y} m^r shews to remain in its present condition.

We conceive one man to have twice \dot{y} power, force, or strength
of another. We therefore consider this quality as capable of being
doubled, tripled &c. In like manner we conceive \dot{y} inherent
force & inertia of m^r as quantities, capable of being doubled,
tripled &c. and thus we make them a subject of mathemati-
cal investigation. But philosophers have frequently too hastily
in thus calling in \dot{y} aid of mathematics in their disquisi-
tions, & have been led into errors. \dot{y}