

That quality may become *it* subject of mathematical investigation, it is not enough *it* it may be greater or less. The affections of the mind have this property of quantity and a firmness for *it* appearance of precision for which mathematics is eminent has led some to make use of mathematical reasonings in comparing them. Thus M^r Huchison says *it* gratitude is directly as *it* product of the favour & *it* dignity of *it* conferrer & inversely as *it* merit of *it* receiver.

$$G \div \frac{FD}{M}$$

and conveys no meaning. Mathematics is not *it* science of quality but of measure. "Now these quantities are reciprocal of measures."

Measurable quantity is of two kinds one may be measured by itself, such as extension, duration, number, & proportion. Other quantity cannot be thus measured of this kind are velocity, density, force. In order therefore *it* these may be made *it* subject of mathematical reasoning they must be invariably connected with some quantity of *it* first kind, in such a manner *it* *it* quantity is always increased & diminished when they are, & *it* each determined degree of this is invariably accompanied by a certain deter-

mined magnitude of the other. In \dot{y} manner motion may be mathematically considered, because it may be accurately measured by space & velocity. How shall we measure force, *vis insita*, *vis inertiae*? By velocity? No; for they are not invariably connected. A body moving to a determined velocity has a determined *vis insita*, but it will not produce such a change of velocity in \dot{y} great as in a small body. We must measure it by \dot{y} quantity of motion. The connection is invariable, for the effect is always produced.

But is there always a determined quantity of \dot{y} one corresponding to a determined quantity of the other? This is so strictly true that we have no notion of any degree of \dot{y} one but by means of the other. Nay we have no notion of any degree of \dot{y} one but in \dot{y} degree of the other. For \dot{y} force of a mass & \dot{y} inherent force of m^r , are not \dot{y} objects of our perception. perhaps they are not even existence, & when we say \dot{y} m^r has an inherent force or an *Inertia*, it is only a short way of saying that, in particular situations, motion is produced, or changed, by it in other m^r .

It is in \dot{y} way only \dot{y} we are entitled to say that the *vis insita* & *vis inertiae* of m^r are equal. We only mean to say in a fewer words \dot{y} \dot{y} changes produced

in \dot{y} moving & resisting body are equal.

As it is only by \dot{y} quantity of \dot{y} effect \dot{y} we judge of the quantity of the power, so it is only by \dot{y} direction of the power. Thus if a body from rest moves in \dot{y} direction **AB** we say that \dot{y} power acts in \dot{y} direction **AB**. In \dot{y} same manner, if a body moving uniformly in \dot{y} direction has its velocity increased without any change in its direction, we say \dot{y} it acts in \dot{y} direction **AB**. If the velocity is diminished, we say that it acts in \dot{y} direction **BA**. If \dot{y} body, \dot{y} by moving uniformly during a very small space of time would have got to **B**, by \dot{y} action of a power is found at **C** at that instant of time in \dot{y} it would otherwise have been at **B**, then drawing **BC** we say \dot{y} \dot{y} power acts in direction **BC**, & take **BC** for its measure.

When motion is observed to have any invariable relation to the situation & distance of any particular body, \dot{y} cause of such motion is ascribed to that body as a quality or power. Such powers are classed under \dot{y} general term of \dot{y} Affections of matter. Thus magnetism, electricity, gravity, are considered as affections of m^r , by \dot{y} it is supposed to produce motion in other bodies at a distance,

altho' we have not yet been able to perceive any material connection between them. We therefore class those affections of m^r & \dot{y} powers of animals along wth \dot{y} powers of m^r hitherto considered, & look on them all as properties of one kind, capable of mutual comparison, by means of their common measure, \dot{y} motion w^{ch} they produce.

However it may appear illogical to compute things so different as impulse & pressure, the impropriety vanishes when we reflect \dot{y} we are only computing their effects.

The exertions of power are very differently termed, according to \dot{y} reference we make of \dot{y} motions w^{ch} are conceived as their effects. If \dot{y} motions are directed to a point, \dot{y} power is termed attraction. If from a point repulsion. These metaphors are borrowed from animal life, and there is no danger of abusing \dot{y} use of them if we always consider them merely as expressive of the effect, & not of the manner in w^{ch} it is produced. were there no changes of motion but such as have been hitherto considering, & are produced by means of impulse, we could have no notion of any powers but inertia & inherent force. But observation points out many w^{ch} have not been reduced to this head.

If a ball lies on a table & I, without striking it only pressed it on one side, it moves when I pull a string fastened to a body, it moves.

If I press or pull wth greater force, it moves faster. If I follow it wth \dot{y} pressure or pull, it accelerates in its motion. When a body is in motion if I press or pull it backwards, it retards, & \dot{y} move, or \dot{y} longer I press or pull it retards the more, & may even stop.

when a stone is no longer supported, it falls, a spring disengaged drives a body before it. Iron approaches or avoids a magnet, and light bodies approach or shun an electrified body. All these changes of motion are also ascribed to powers called gravity, elasticity, magnetism, electricity, & these, to distinguish them from impulse, are called pressing powers. Farther such powers are suggested, even without any production of a change of motion. when I wrestle wth another, & feel \dot{y} in order to prevent being thrown I must exert power. This is also transferred by analogy to m^r , & when known power produces no motion, it is conceived as apposed by another power. The vis inertia of **A** may not only be deduced from the change w^{ch} it produces on a moving body **B**, but also from the

change in A's motion not being such if B shall suffer
no change. Without considering if B moving if the
velocity 2. suffers a diminution 1 & may only observe if
B does not acquire if change 2. In like manner pressing
powers are deduced not only from the changes if they
produce but also from if changes if they prevent.

Thus if cohesion of m^r in a string is deduced not
only from giving motion to a body if I pull, but
also from preventing a suspended heavy body from
falling.

The solidity of m^r is not only deduced
from if rest produced in if body if strikes it but
from supporting if weight if lies on it.

When powers are conceived as residing in m^r
& if effect as produced by their exertion, then if body
conceived as possessing if power is said to act on the body
in if if effect is perceived.

No term in Nat. philos. is used in a more loose
& undetermined sense than if term action.

It is a term borrowed from animal life.

A man exerting his powers in strict language is
said to act. The mind is immediately employed on

if body, & properly speaking, acts only on it. But as its
action on if body is only in a view of producing some effect
on external things, we overlook if instruments and atten-
tive only to if ultimate object. are said to act on other bodies.
In if strict sense of if word therefore activity means
the production of motion where there was none before
either in if agent or thing acted on; an active being is one
if can do this.

From if by analogy we have transferred if Idea & if
term to if production of motion in one body by means
of motion in another, & if moving body is said to act on
if in if if motion follows. Further, as we find ourselves
obliged to act in order to diminish or destroy motion
so a body is analogically said to act when it dimini-
shes or destroys if motion of another body.

A man is not said to act unless he produces some
effect. Thought is the effect of if thinking principles
acting, and motion is if effect of if mind's acting on the bo-
dy. If we attempt to fix our attention neither on if mind,
nor on if effect, but on the procedure in producing it, we
have no object of contemplation.

The exertion is an event & is not *ij* object of perception. when therefore we speak of action as distinct from *ij* agent we usually mean nothing but *ij* effect. Sometimes indeed we mean *ij* fact of producing it, but *ij* is a very obscure conception. There is action therefore only in so far as we conceive *ij* production of an effect. It requires a violent action in me to begin motion in a slide; but I never conceive myself as acting while sliding along. In like manner action is conceived as necessary in order to begin motion in *m^o*; but none to continue it. A body in motion therefore is not acting, for it is producing no effect. Its own motion is not *ij* continual production of an effect, but *ij* continuation of an effect already produced. We indeed conceive motion as *ij* effect of an action but there would be no effect if *ij* body were not moving. Motion therefore is not action, but *ij* effect of an action.

Action therefore must be restored to *ij* production of a change of motion, & is perceived by us in no other way than *ij* change produced. It is therefore an inaccuracy of analogy to say *ij* in a moving body there is a power of preserving motion. There is indeed a power of changing *ij* motion of another body, but not yet exerted or brought into action.

Action is sometimes considered as a quantity & made *ij* subject of mathematical investigation.

This obliges us to agree on a measure of action. The change of motion is a proper measure both of *ij* power & *ij* action, and is indeed *ij* only thing usually meant by either term.

The actions of animals have different names. If in boxing I strike my antagonist, I am properly said to act. But if I only parry his blow I am said to resist tho' *ij* exertion of my strength is *ij* same. This distinction is also analogically applied to *m^o*. When a moving body changes the motion of another & *ij* action is mentioned in reference to *ij* change it is called action, impulse. But when considered in relation to its maintaining its own condition, it is called resistance. In *ij* sense both a moving and a resisting body are said both to act & to resist each other & any change of motion.

Sir I. N. considering *ij* there was in both cases *ij* same exertion of power, called resistance by *ij* more abstract term reaction. The distinction therefore between action, or impulse, & reaction, or resistance, is a distinction not in *ij* thing, but in *ij* reference to we make of *ij* event, as changing *ij* situation of other body, or maintaining its own.

Since then *ij* measure of *ij* action or impulse & of *ij* reaction or resistance is one & *ij* same, namely by the effect or change produced, & since in

all communications of motion by change is equal & opposite
we infer that in the communication of motion
by impulse, action is equal & contrary to re-
action.

with respect to pressing powers, we also call
their exertion action. Gravity, Electricity &c are said
to act on bodies. This action is called by its distinctive
name of pressure, & its opposite action is termed Resistance.
But these are distinctions not in the thing but in
thought. When my strength is exerted in order to
throw my antagonist to whom I wrestle, I act; but when
in order to prevent his throwing me, I resist. They are
best distinguished by this, that when, upon one power ceasing
to act, motion follows, its action of its other power is called
action. When motion does not follow, it is called resistance.
Thus when a string breaks & a body falls, Gravity is
said to have acted: when I cease to press a body to a table, &
its body still lies there, its solidity of its table is conceived as
only to have resisted. We also conceive reaction to take
place in such opposition of pressing powers. we know
pressing powers are greater & less, but when no change of
motion ensues from their exertion, we have no measure
of their quantity. Were an obstacle immovable, no

change of motion produced in its impelling body would
give us any notion of its quantity of its resistance, tho'
it would of its action of its resistance. But if known
quantities of motion are just suffering to overcome them
or known powers just suffering to ballance them, we
have proper measures of their quantity. Thus I call
its cohesion of its string or its table double it will just sup-
port a double weight, or just extinguish a double quantity
of motion. Since then these effects are its only way in it we
conceive its existence of such powers or measure their quantity
we must affirm its in its mutual actions of
pressing powers action is equal and
contrary to reaction.

But it remains a question to be decided not from
metaphysical principles but from experiment, whether in
all its exertions of a pressing power there is a concomitant
exertion of an equal & contrary one. This must be consid-
ered as already demonstrated in all those in ballance
each other & in all those in tho' they do not ballance
each other yet oppose each others exertions in pro-
ducing motion. For as it only from its ballancing a
known power, in whole or in part that we judge of its
existence of its other it can only be by its quantity.

is it ballanes if we can judge of its quantity. Accordingly
it is confirmed by experiment a peice of clay is I press to
my table with my finger is equally flattened on both sides.

But it does not necessarily follow it in cases where
motion in it one is not opposed by motion in it other
there should always be an equal & contrary action.

Sir J. N. thinks it does, & he gives following demon-
stration. Suppose **A** attracts **B**, but if **B** does not
attract **A**. interpose some obstacle it will be uniformly
pressed, & will move to it side where it is not
pressed. And as it inequality of pressure still con-
tinues, it will go on in it direction to an accelerat^{ed}
velocity. is, says he, is absurd, & contrary to it first
law of motion.

But there is no absurdity in any matter of fact,
if it does not involve a contradiction. There is there-
fore no absurdity in saying that a body when left
to itself should begin to move, & go on accelerating
forever. Some of the instances too is are adduced in
support of it law are not proper. A man in one boat
pushing or pulling another boat makes both recede
or approach without equal quantities of motion.

But in it case if man either pushes one boat with

his hand & it other to his feet, or pulls on boat to his
hand & pushes it other to his feet. All other instances
where there is a material connection between it
bodies is acquire equal & contrary quantities of motion
are of it same kind.

There are here other experiments & observations is are
not liable to it objection. Magnets & iron mutually ap-
proach & recede. So do light bodies & electrified bodies

The same is perceived between it Earth & Moon, &
causes it tide. The same will be afterwards shown
of it Earth, Moon & sun, & in short, between all it
bodies of it solar system. It will be shown it there is
a mutual tendency of all it planets to it sun and
of it sun to all it planets, & it all mutually tend
to each other, & from it tendency they are all disturbed
in it regularity of their motions. It also appears from it
observations of Messieurs Buger & Condamine in Peru,
& of Maskelyne in Scotland it it different parts of this
globe tend to each other, & by it mutuall tendency
are kept together.

Since then it fact may be affirmed to take place
invariably in all it exertions of pressing powers
which we can observe, we may now assume as it
second general law of motion that

Every action of bodies is accompanied by an equal and contrary reaction.

For γ resolution of forces see Book
from page 21 to page 38.

Such then are γ general laws of motion, γ principle from γ we are afterwards to reason synthetically, in deducing γ subordinate laws of motion, in investigating γ powers & in explaining γ phaenomena of nature. motions are either free or constrained - a body is said to move freely, when it moves in γ path, & γ velocity is its own inherent force & the acting powers would determine. Thus γ planets, terrestrial projectiles move freely. A motion is constrained when it is not allowed to follow γ path along γ its own inherent force or γ action of γ sollicitating power would determine it to move. Thus a body descending along an inclined plane, or swinging by a string has its motion constrained.

It is evidently proper to consider γ free motions of bodies first, as it is from them γ we get our knowledge of the active powers of nature.

Secondly not only single bodies move but also col-

lection of bodies have besides their particular motions, a motion γ belongs to γ whole assemblage. Thus besides the diurnal revolution of γ Earth round its axis and the revolution of γ moon round γ Earth, the assemblage has a general motion round the sun. In like manner that assemblage of m^r called a ship has a particular motion of rolling pitching, of its several parts & the general progressive motion in its course. It is evident γ these must be complex, and therefore to be postponed to γ motions of single bodies. Such an assemblage of bodies, connected together by powers of mutual tendency or by γ cohesion of m^r I shall call by γ general name of a system.

Lastly motions may be uniform or varied. As our original notions of bodies are taken from uniform motion, & as every variety is ascribed to some power as its cause it follows γ uniform motions are to be first considered, as γ foundations of our future reasonings.

I shall therefore in γ course of Dynamics consider first γ free & secondly γ constrained motion of single bodies & then γ free & constrained motions of systems, in each of γ classes I shall consider first γ uniform motion, & then γ varied & take the varieties in γ order of their simplicity - first -

of the free motions of a single body — see book from page 47 to page 50 —

I of uniform Motion.

we have already seen *ij* motion is of its own nature uniform, & therefore when a body is in motion & affected by no external cause, its motion will have *ij* properties formerly demonstrated of uniform motions.

I. A body therefore in motion must, if left to itself, describe equal spaces in equal times.

II. A body in motion must describe spaces proportional to *ij* times.

III. In different uniform motions

$$S \div TV$$

$$T \div \frac{S}{V}$$

$$V \div \frac{S}{T}$$

In all these cases our determinations proceed on the composition of *ij* quantities & have the proportions of *ij* times with quantities *ij* have the proportions of the times with

quantities *ij* have *ij* proportions of *ij* velocities. The proportions of *ij* spaces are easily had, because *ij* spaces are *ij* objects of our perception. But how shall we determine *ij* proportions of the times & how are they rendered capable of a composition. This is a question of the greatest importance in Nat. Philos. for without a knowledge of the proportions of times we can't pronounce any thing with regard to motion *ij* will be susceptible of any application.

As *ij* proportions of time are not immediately perceivable of themselves. The only method *ij* we can employ is to find out a proper measure of its parts, some proper quantity so intimately connected to it as *ij* we can demonstrate *ij* they increase & decrease together, & *ij* a certain determined quantity of *ij* one always corresponds to a certain determined quantity of *ij* other. This measure must be well known in order to be usefull, & *ij* comparison must be simple. From *ij* it follows *ij* uniform motion is the simplest measure of time. on *ij* one hand the properties of the parts of a straight line is what of all others we conceive to *ij* greatest distinction & farther there is no proportion so clearly perceived as *ij* of equality. now in uniform

motion the proportion of the parts of time is equal to the proportion of the parts of space run over in them.

Uniform motion therefore gives us at once \bar{y} means of composing the proportions of \bar{y} parts of time to \bar{y} set of proportions \bar{z} we understand best of any, & also to do \bar{y} in the simplest manner.

Moreover, independent of its simplicity, it is \bar{y} most most natural measure we can employ; for there is no proportion \bar{z} we know so accurately as \bar{y} of the parts of space, & in general, any motion, whose law is given, would lead us to a discovery of \bar{y} proportion of \bar{y} parts of time by means of \bar{y} relation which subsists, in consequence of \bar{y} law between \bar{y} spaces & times. If therefore there is any kind of motion, in \bar{z} \bar{y} analogy between \bar{y} time & space is given, independent of any hypothesis & by \bar{y} very nature of motion, & if \bar{y} kind of motion is \bar{y} only one in \bar{z} \bar{y} analogy is given independent of any hypotheses, it is necessarily the most natural measure of time.

Now it is only uniform motion \bar{z} has \bar{y} character. For motion is of its own nature uniform, & it is accelerated or retarded only by \bar{y} actions of external cause, in \bar{z} case it is susceptible of an infinity of laws of variation. No varied motion therefore becomes \bar{y} proper measure of time till

we have ascertained \bar{y} law of its variation or its difference from uniform motion, & \bar{y} supposes us already in possession of the measure of time.

But how shall we know whether any motion is uniform or varied, \bar{y} we may employ or reject it. In answer to this I observe \bar{y} we are equally ignorant of \bar{y} law of all motions \bar{z} are not uniform, & \bar{y} result is that we cannot know exactly, & \bar{z} \bar{y} ultimate strictness \bar{y} proportion of \bar{y} parts of time. But still uniform motion is \bar{y} proper & simple measure of them. Not being therefore able to find an accurate measure of time, we must endeavour to find one \bar{y} will be nearly so among \bar{y} motions \bar{z} we judge most uniform.

we have three ways of judging of \bar{y} uniformity of motion.

1. If a body moves over equal spaces in times \bar{z} we have reason to think equal, \bar{y} is in \bar{z} repeated experiment, when \bar{y} equal effects are produced \bar{z} we have reason to think should employ \bar{y} same time. The time of emptying a sand glass is reasonably supposed by us to be always the same.

If therefore in these times \bar{y} Sun is observ'd to move thro' equal arches, we have reason to think his motion uniform. Accordingly \bar{y} was \bar{y} first invention for measuring time & tho' daily motion of \bar{y} sun being found always to

employ if same number of intervals of emptying if glafs,
was substituted as a proper measure. For besides ine-
qualities w could be observed in different glafes, the obser-
vations made to them at different places could not be com-
pared wout refering them to some common measure.
Thus if measures were changed, & whereas a day was for-
merly counted so many glafes, a glafs was afterward counted
such a part of a day.

2. When we have reason to believe if if action of any
accelerating or retarding cause w we know however to what
is constant. It is in if manner, by uniting these two
methods, if if motion of if Earth round her axis is reckoned
if most accurate measure of time; & if supposition is
not only not contradicted by if other ultimate motions,
but found perfectly to agree w them. we have found
reason for rejecting if Sun's diurnal & annual motion
because we have discovered if existence of accelerating & retard

X

retarding causes is are too considerable not to produce sensible irregularities. Whereas in \dot{y} motion of \dot{y} Earth, all \dot{y} motions is are performed on its surface, & it must indeed disturb its regularity, & \dot{y} motion of \dot{y} tides, cannot produce a deviation \dot{y} will bear any sensible proportion to the motion of \dot{y} Earth.

3. When we compare \dot{y} supposed uniform motion to other motion & observe the same law in both - Thus if we see two bodies move in such a manner \dot{y} \dot{y} space \dot{y} they describe in \dot{y} same time are always in a given ratio and \dot{y} \dot{y} holds whether \dot{y} bodies have both begun to move at \dot{y} same time or at different times, & when we can see no cause of their motion but \dot{y} motion once begun then we reasonably judge these motions to be uniform. All these circumstances united have led us to prefer \dot{y} motion of \dot{y} Earth round its axis as \dot{y} proper measure of time. Subdivisions of a revolution are had by means of \dot{y} fixed stars, which, as they do not change their relative places are proper measures of \dot{y} parts of space. By this means we are enabled to discover \dot{y} \dot{y} motion of \dot{y} sun & moon are not uniform & to determine \dot{y} laws of their variation, & thus arrive at a knowledge of \dot{y} changing forces the principle object of nat. philosophy. For varied motions & see book from p 53 to p 67

Theorem Fig. 13

Let a body A move from rest along AC by an accelerating force directed to C. Let there be a curve KDF such that the area ABDK is always proportional to the square of its velocity w^h its body has acquired at B. Then I say its accelerating force at any point B is always proportional to the ordinate BD.

Demonstration

Take BE an infinitely small part of AC, & draw the ordinate EF (all its space AB^s, and its area ABDK v²). Then BE = s, & BEFD = 2vv, and BD = $\frac{2vv}{s}$.

Now F = $\frac{v}{s}$

and s = vt

Therefore F x s = $\frac{v}{vt} \times vt = v$

and F = $\frac{vv}{s}$

But BD = $\frac{2vv}{s}$

Therefore BD = 2F

and BD is always \div F

Coroll^y If v² \div s, then its power is constant, for then A \div X² & consequently v \div X, \div 1.

Theorem

Let a body A move as before, & let BD be always proportional

to its force at any point B. Then the area ABDK will always be proportional to its square of its velocity at B.

This is its converse of its former, & is demonstrated in its very same way, only reversing its steps.

Corollary 1. Then its square root of $\sqrt{\text{ABDK}}$ is equal to its velocity acquired by moving thro' AB.

Corollary 2. Draw another ordinate MN. Then MNDB is equal to its square of its velocity acquired at B. the square of its velocity acquired at M; and its square root of $\sqrt{\text{MNDB}}$ is equal to its increment of its velocity acquired in moving over the space MB.

Corollary 3. If F constant then its space is proportional to its square of its velocity. For then v \div 1, \div X; and A \div X², but A \div v², or S \div v².

Theorem

Let a body A move as before, & let there be a curve LHI such that its ordinate BH is always inversely proportional to its velocity acquired at B. Then the area ABHL will always be proportional to the time of moving from A to B.

Demonstration

In AC take an infinitely small part BE and draw the ordinate EI

When $BH = \frac{1}{2} S$

and $BE = \frac{1}{2} S$

and $BEIH = \frac{1}{2} \times \frac{1}{2} S = \frac{1}{4} S^2$

But $v = \frac{S}{t}$

Therefore $v^2 = \frac{S^2}{t^2}$

and $t = \frac{S}{v}$

or $t = \frac{S}{v}$

and $\int t, \text{ or } t = \int BEIH \text{ or } ABHL$

Corollary 1, If F is constant then $S \propto t^2$. For then $x \propto v^2$, then is $x \propto \frac{1}{2} v^2$, or $v^2 \propto \frac{1}{2} x$, or $v \propto \sqrt{\frac{1}{2} x}$, or $v \propto x^{\frac{1}{2}}$; therefore $A \propto x^{\frac{3}{2}}$, or $x \propto A^{\frac{2}{3}}$.

II. If F is constant then $v \propto t$. For then $S \propto v^2$ and it has just now been shown $\frac{1}{2} S \propto t^2$ therefore $v^2 \propto t^2$, and $v \propto t$. Such a force therefore produces an uniformly accelerated motion. For if $v \propto t$, then $v \propto t$.

III. If F is constant, then if time of describing any given space a is double of the time in which space would be uniformly described with velocity v acquired at its end of its accelerated motion. Here v is also supposed known, & $\frac{a}{v}$ is the time T of the uniform motion. Now since v is given a is given, for $a = \frac{1}{2} v^2 T$. Then $A = 2a \frac{1}{v}$, or $2a/v$. But $A = t$ therefore $t = 2a/v$, or $t = 2T$. Hence if any space a is described by the action of a constant force F in a time t , twice of space will be described

in the same time uniformly with the velocity acquired at its end of its accelerated motion.
Cor. Draw any other ordinate MO then $MBHO$, time of moving from M to B

Theorem
The spaces thro' which any two constant forces F and Q must impell a body in motion to communicate it same velocity v are reciprocally as the forces.
Let $AB, \alpha B$ be spaces, & $AF, \alpha Q$ be forces. Then if curves expressing the law of the forces are straight lines FD, QE . The velocities are supposed equal therefore the areas $ABDF, \alpha BEQ$, are as the squares of the velocities must be equal. Therefore $AF : \alpha Q = \alpha B : AB$.

Of Curvilinear motions
Since a body of itself tends to move in a straight line, it cannot move in a curve without the continual action of some transverse force whose action, being combined with its inherent force of the body, makes it continually to deflect from the rectilinear path, in every moment

tends to pursue along the tangent to its point of the curve where it is.

OH The curvilinear motions therefore are always indications of its continual action of a pressing power. The degree or quantity of this pressing power is to be deduced from its effect it produces, it is from its deflexion which at occasions from a straight line this again must be measured by its distances of its point where its body really is from that point of its tangent to its curve to which it would have arrived in the same instant had its pressing power not acted on it.

The deviation of its pressing power will be deduced from its position of its line joining these two points.

This line is in all cases its subtense of its angle of contact at its point from which the deflections is supposed to begin.

But as its pressing power is unknown, & its exertions may vary according to any law and as its deflexion from the straight line at its end of any given time, or at the end of any arch of the curve, is its effect of its accumulated, & perhaps varied action of its power, it is evident that it will not give us a precise notion of its exertion of its power at any instant of its action.

It follows then that its only way of determining its accuracy is

to consider its subtense of its angle of contact in its nascent or evanescent state, when its body has described a nascent arch of its curve, in its case its ratio of its inherent force to its transverse force is its same to its ratio of its nascent arch to its nascent subtense of its angle of contact, is again is its same to its ratio of its nascent chord to its nascent subtense of its angle of contact.

From all this it follows that our investigation of those powers of nature to produce curvilinear motion must proceed on its fluxionary properties of its curve described.

On its other hand suppose its nature of its transverse force to be known we can at all times tell what curvilinear motions will ensue upon its acting on a body moving in any known direction to a known velocity. For we know its degree of its force, we know its motion to which it will produce on a given body, & consequently we know its proportion between its proportion between its motion & its of its body. That is we know its ratio of its nascent subtense of its angle of contact to the nascent chord of its curve along which it tends at its moment to direct its body. Farther if we know its direction of its transverse force we shall know its position of its subtense nothing more is necessary to determine any curve whatever.

1. In general we can affirm that its body will deflect continually & its path will be a curve.

2. Its path will be concave to its side to which force is directed.
3. Its deflection from its tangent will be so much greater as its ratio of its transverse force to its inherent force is greater.
4. Its deflection will be greater as its transverse force is more nearly perpendicular to its direction of its bodies motion.

Instances of free curvilinear motions are frequent, & of great dignity. all bodies projected on the surface of the Earth in directions oblique to the horizon — all the planetary motions — Therefore they deserve our accurate examination, as they serve for discovering the greatest powers in nature.

However variable these curvilinear motions may be, there is one general principle in which they all agree, & by it we are enabled in case to investigate the nature of the transverse power which produces them.

If a body A revolves in any curve ABC (fig. 14) & if it be found from observation that there is a power S such that, drawing from any three points A, B, C , the straight lines AS, BS, CS , the areas SAB, SAC are proportional to the times of describing the curvilinear bases AB, BC ; then the transverse power which causes the deflection from its rectilinear motion is a power continually directed to the point S . Since areas are described proportional to the times equal

areas are described in equal times.

Let therefore AB, BC be two nascent arches described in two equal times. The arch AB may be conceived as ultimately coinciding with its tangent ABD . Take BD equal to AB draw BC, DS , & BE equal & parallel to DC , & join CE . DE indicates by its direction the direction of the force which is acted at B . For if the body being actuated by its perseverance only would have gone on to D in a time equal to that of describing AB , that is, in the time of describing BC , BE therefore is the direction of the transverse force acting at B . But since AB is equal to BD the triangle SAB is equal to the triangle SBD . But SAB is equal to SBC therefore SBC is equal to SBD , & DC is parallel to BS . Therefore the line BE lies in the line BS , & the transverse force at B is directed to S .

On the other hand if it is previously known that the transverse force is always directed to S , it may be demonstrated that the body will describe curvilinear bases of areas proportional to the times in which the bases are described. The demonstration is just the inverse of this. These two theorems were discovered by Sir J. N. & make the foundation of all his discoveries in the just movements of Nature. The first serves for the discovery of the determining motions occasioned by the forces already discovered.

For, in its first place, you must observe that altho' its measure of the transverse force is a deflection from the rectilinear motion, yet it cannot be discovered till its direction is known.

Secondly knowing its property of its areas of its times, we can at any time assign its power in a revolving body is to be found, by means of its known motion during any given time. Suppose, for instance, that its planet Mars acted on by a force directed to the sun, has in one day described its arch **AC**, & I want to know in what point of his orbit he will be after three days. Draw a line **SG** making its area **CSG = 3ASB**, and **G** is its point of his orbit to which he will arrive at its end of three days.

When its directions of transverse forces always pass thro' a given point, they are called its central forces, & its point is called its center of forces; if they are always directed to its point, & its curve is consequently concave to its point, they are called centripetal forces, & by analogy attractions. If they are directed from its point, & its curve is convex to it, they are called centrifugal forces, & by analogy repulsions. Instances of both occur in Magnetism & Electricity.

Of Central forces

If a body revolves in a curve by means of a force directed

to a point, its velocity is always inversely proportional to a perpendicular let fall from its centre of forces on its tangents drawn thro' its place of its body. For fig. 15 let **AB, CD** be two nascent arches described in equal times; they may be considered as coinciding wth its tangents **BH, DG**. Since its triangles **SAB, SCD** are equal, their bases **AB, DC** are reciprocally proportional to their heights **SH, SG**. But as their bases are described in its same time they are as its velocity w^{ch} they are described: therefore its velocity at **A: veloⁿ at C = SG:SH**

If a body actuated by a centripetal force move in any curve, & another body actuated by its same force moves in its line passing thro' its centre of forces, & if their velocities at any equal distances from its centre be equal, they will also be equal in all or equal distances from it.

Let its straight line **AS** pass thro' its centre of forces, & let the bodies have equal velocities in **I** and **D** equally distant from **S**. Join **IS**, & taking **DE** a small part of **DS** describe its circle **ENK**. Draw **KS**, and draw **NT** perpendicular to the curve in **T**.

Since **DS = IS**, its centripetal forces at **D** & **I** are equal. Let them be represented by its equal lines **DE, IN**. Resolve **IN**, into **NT**, w^{ch} produces no change of velocity along its curve, &

IT, to acceleration of velocity in it. The accelerations in equal moments of time are proportional to the accelerating forces. Therefore this acceleration in D is to that in I—DE:

IT, but in different small times the accelerations are as the forces & times jointly for in small times the forces may be considered as constant. But since the velocities at A and D are equal the times of describing DE and IK will be as DE, IK therefore the acceleration in D is to the acceleration in I as $DE \times DE$ or DE^2 , or IN^2 : $IT \times IK$. But since the angular velocities are inversely proportional to the square of the distance from the centre of forces for the angular velocities at A & C (fig. 15) are as the increments of the angles ISA, ISC it was formerly demonstrated to be as $\frac{AE}{AS} : \frac{CE}{CS}$. Now since $AE \times AS = CF \times CS$ on account of the equality of areas then $\frac{AE}{AS} : \frac{CF}{CS} = \frac{AE \times AE}{AS^2} : \frac{CF \times CF \times CS}{CS^2}$. But since $AE \times AS = CF \times CS$, then $AE : CF = CS : AS$ and $AE^2 : CF^2 = CS^2 : AS^2$ therefore the angular velocities are reciprocally proportional to their distances.

An Orbit lies all in one plane
It has been shown that the nature of the central force was to be deduced from that of the curve in which it incited the body to move

now these are susceptible of an infinite variety, & there is no occasion for examining what kind of central force would make a body describe each. There is no way of connecting them so will enable us to deduce a general method of investigation applicable to all.

If a thread revolves round a fixed point in a plane, it will describe a circle. If it gradually untravels from a cylinder it will describe a spiral. If it untravels from a cycloid it will describe an equal cycloid, and so on of other curves. Every curve will produce another curve by a thread gradually untravelling itself from it. on the other hand there is no curve, concave to one side, in all directions will be but what may be described by the gradually untravelling of a thread from some other curve. Every curve may in this manner be considered as the evolute of some other curve Fig. 16.

Now while the thread is unfolding, it may, in every moment of its motion, be considered as the momentary radius of a circle, whose centre is that point of the evolving curve which is just going to quit. The extremity of the thread therefore is at the same time describing the nascent arch of the curve called the evolute, and the nascent arch of the circle. Consequently these two nascent arches coincide.

From of its genesis of its curve it appears, if any body, while it describes its nascent arch of the curve, may be considered as describing its nascent arch of its coincident or equicurve circle. If therefore we can find a general expression for its central force directed to any point of a circle, we at the same time find its expression for its central force in a coincident arch of a curve, where its line drawn to the centre of forces is similarly placed to respect to the tangent. & all that now remains is to determine from its properties of its curve its chord of its equicurve circle.

Let two bodies A, F / Fig. 17 / revolve in the circles ABE, FGK by means of central forces directed to any points S and T , and let AB, FG be nascent arches described in equal times — draw its chords ASE, FTK , and its chords AB, FG, BE, GK and its tangents AC, FH — and draw BC, GH parallel to AE, FK , and BD, GI parallel to AC, FH —

It is evident its CB or AD is its proper measure of its centripetal force at A , & its FI is its measure of its centripetal force at F .

By its property of its circle $EA:AB=AB:AD$, and $KF:FG=FG:FI$ — therefore

$$AD = \frac{AB^2}{AE}, \text{ and } FI = \frac{FG^2}{KF}$$

But since its arches AB, FG are described in equal times, they

are as its velocities at A & F , and AD, FI are as its forces at A & F and AE, KF are its chords at A & F — therefore

$$F, A : F, F = \frac{V, A^2}{CA} : \frac{V, F^2}{CF}$$

Hence we deduce its following general theorem and formula —
Central forces are directly as its squares of its velocities, & inversely as the chords of its equicurve circle passing thro' its centre of forces —

The formula is $F = \frac{v^2}{C}$
I (Cor¹) — If S is its centre of its circle then C is equal to twice its radius. But as halves have its same proportion to their wholes C will always be proportional to R , or to the distance D from its centre of forces —

2. The velocity will be every where equal, & its motion uniform. For its perpendicular to its tangent, is always reciprocally proportional to its velocity, is always its same or radius —

3. In different circles its central force is directly proportional to its distance and inversely proportional to its square of its periodic time P .

For the velocity is directly as its circumference described

and inversely as time of describing it. or $v \div \frac{D}{P}$. because
 D or radius \div to its circumference. Therefore $v^2 \div \frac{D^2}{P^2}$, and since
 $F \div \frac{V^2}{C}$, and $C = D$, then $F \div \frac{D^2}{P^2}$, or $F \div \frac{D}{P^2}$.

Cor^y 4. If a body fall along its radius with its uniform
 action of its force it retains it in its circumference, it will
 acquire a velocity equal to that it is it resolves in its cir-
 cumference. Let AL be its space thro' w^{ch} it must fall in
 order to acquire its velocity of its revolution, and let AF be the arch
 described uniformly during its time T of falling along AL.

Then $AF = 2AL$, and $AF^2 = 4AL^2$. Let AB be a nascent
 arch, and AD be its versed sine, or its space thro' w^{ch} its centripetal
 force would cause its body to fall in its time t of describing
 AB. Then we have its proportion $AD:AL = t^2:T^2$, or
 on account of its uniform motion in its circumference

$AD:AL = AB^2:AF^2 = \frac{AB^2}{AG}:\frac{AF^2}{AG}$. But it has been
 Demonstrated its $\frac{AB^2}{AG} = AD$. Therefore $\frac{AF^2}{AG} = AL$, and
 therefore $\frac{4AL^2}{AG} = AL$, or $\frac{4AL}{AG} = 1$ & therefore $AG = 4AL$,
 & $AC = 2AL$.

Cor^y 5 The periodic time is to its time of falling thro' AL as

its circumference of a circle to its radius. For $AF = R \cdot \sqrt{P:T}$,
 $AF = AFA:AF = \text{circumference}:\text{Radius}$. But $T, AF = T, AL$
 therefore $P:T, AL = \text{Circumference}:\text{Rad.}$

These two formulae $F \div \frac{v^2}{C}$ & $F \div \frac{D}{P^2}$ are all it is
 necessary for investigating its central forces which produce
 any motions w^{ch} we can observe. The first serves for dis-
 covering its law of its central forces producing a revolution in
 any one orbit and its seasons for comparing together the
 forces w^{ch} produce revolutions in different orbits.

Let it be always remembred its in order to avoid error,
 it is proper to consider F not as its immediate symbol
 for any metaphysical being existing in its body or the
 agent, but as a symbol expressing merely its quantity
 of approach w^{ch} its revolving body makes to its centre of
 forces in some small given time, suppose a second.

This way of considering it has also its peculiar advan-
 tage its we can compare every accelerating force whatever
 w^{ch} a central force is an accelerating force whose direction
 is transverse to its motion of its body w^{ch} the fall of heavy
 bodies let us take for its term of comparison a second.

In this time a heavy body falls thro' 16. feet. If there-
fore F is taken as a symbol of its effect of gravity $F = 16.1 \text{ feet}$.

Let us express this however by its letter G . G then is the
measure of gravity considered as an accelerating force, & its
proportion of G to $\frac{V^2}{D}$ will always express the proportion of
gravity to the accelerating force, or $\frac{V}{DG} = F$, that is whatever
is its nature of its centripetal force, its difference of its forces to
is a body would revolve in ^{atque} a quiescent curve, & another body
revolve in its same curve, while it revolves uniformly round
its centre of forces, are in its inverse triplicate ratio of its dis-
tance from its centre, that is to say $\frac{V^2}{DG}$, or $\frac{1}{16.1} \frac{V^2}{D}$ will
be its number of feet to the central, or other accelerating
force F will cause the body to move thro' in a second.