[C]onstrained motions.

[A B]ody in motion may meet with obstacles wh will totally [d]estroy, or wh may diminish, or otherwise change its motion. Obstacles wh destroy may either be quite invincible, so as to destroy any motion whatever, or they may be just able to destroy ye motion under consideration. An invincible obstacle too may be of such a nature as not to permit any motion to begin, as when a body pulls by an inflexible line fastened to a fixed point. or it may be such as only to allow a motion in its own direction.

1. If a body in motion meets an immoveable and impenetrable plane at right angles, it will rest. For it must either move to one side or return back. There is no cause to determine it to either side, therefore it must either return back or rest. It cannot return back, for it must previously rest and once it is at rest, it cannot move, unless some moving force act on it, wh we do not suppose to take place in the present case.

2. Since then an immoveable plane destroys motion it must be considered as exerting a power measured by ye change of motion wh it produces; and ye direction in wh this power is exerted is to be esteemed in ye direction of ye change

is it produces, that is in a direction tending ...
proportional to it- this exertion is resistance. ...
conclude ye the resistance of resisting planes is perpen...
from their surface ~

3. If a body A (fig. 19) moving in the direction AB meets an immoveable plane BD, inclined to its direction in ye angle CBD, it will move in ye direction BD with a velocity is is to its former velocity as Cosine CBD to Radius, and ye loss of velocity is it sustains will be proportional to ye versed Sine of that angle ~

At ye point B ye body is acted on by two forces - its inherent force in ye direction BC, & ye resistance of ye plane in in ye direction BE perpendicular to BD. Draw CD parallel to BE, and DE parallel to BC. Then it is clear that BD is the the motion is must result from ye joint action of ye forces BC, BE. This motion is uniform, for ye action of ye resistance is instantanious. BD therefore is to BC as ye velocity after ye stroke to ye former velocity. But if BC is Radius then BD is ye Cosine and DF ye versed sine of ye angle CBD ~

Lemma (fig. 19 No 2)

The versed sine of ye angle of contact is an infintesimal of the

A ... About ye point of contact A is the chord AB ... radius describe ye arch of a circle BC cutting ye tangent ... C; draw CD perpendicular to AB.

Then we have AB:BC=BC:BD.

Let B continually approach A & at last unite to it. Then BC is infinitely small or vanishes in respect to AB - and BD is infinitely small in respect of BC ~

4. If a body A, moving in ye direction AB, (fig. 20) meets a curve channel BCD, in whose plane it lies, & is AB touches in B, it will move in ye path BCD is an uniform motion, and will lose none of its velocity ~

For its loss of velocity in each nascent chord of the curve is as ye versed sine of ye angle of contact that is an infintesimal of the second order. an infinity therefore of such diminutions will make but an infinitely small part of its velocity. It therefore moves on is an undetermined velocity, and its motion is therefore uniform.

N. B. This may be rigorously demonstrated as follows (fig. 21)

Lemma

The subtense of ye angle of contact in any circle is directly as ye square of ye nascent arch & inversly as ye Radius ~

This is demonstrated in the last propo[illegible]
forces [Fig. 17] when it was shewn AD [or [illegible]
making AE the diameter then ye radius is equ[illegible]
AE & BC $\div \frac{AB^2}{R}$

5. Let now a body move by its inherent force along ye curved channel ABC [fig. 24] in every point B it endeavours to proceed in ye direction of the tangent BD, & if AB, BC are two equal nascent arches, & BD = BC, then DC will be ye effect of ye resistance canal to ye rectilineal motion along BD. But ye resistance was exerted at ye point B, & therefore in a direction BE perpendicular to ye tangent BD. Now if ye curve ABC be conceived as produced by ye evolution of some other curve, BE will be ye position of ye momentary radius while it was describing ye nascent arch AB. Let E be ye momentary centre, or the centre of ye equicurve circle. Then BD is ye subtense of ye angle of contact and therefore $\div \frac{AB^2}{R}$. But CD = BF ye resistance of ye curve. now ye resistance is exerted against an equal & contrary pressure by wch ye body tends to recede from the centre E of ye equicurve circle. and wch may therefore be called ye centrifugal force of the body, arising from ye curvilineal motion.

The versed sine of ye angle of [illegible]

[illegible]fugal force therefore is always proportional to [illegible] it is always equal to $\frac{AB^2}{2R}$. But AB will represent [illegible] velocity, being $\frac{S}{t}$. Therefore ye body in its motion along ye curve canal always presses it to a force $\div \frac{V^2}{R}$. or ye centrifugal force always is proportional to $\frac{V^2}{R}$

N. B. It is easy to see that ye law of ye centrifugal force is precisely ye same wch ye formerly demonstrated of centripetal forces. wch were formerly demonstrated to be directly as ye square of ye velocity, and inversely as ye chord of the equi-curve circle, passing thro ye centre of forces. In ye present case this chord is always the diameter because the resistance of ye canal is always perpendicular to the tangent, & therefore its direction always passes thro' ye centre.

It appears therefore yt ye resistance of ye solid canal performs ye power of a centripetal force. wth ye sole difference that it is not directed to a fixed point within the curve, but always to ye momentary centre of ye nascent arch, i.e. it is always directed to a point in ye evolving curve by wch evolution it is produced.

All ye analogies therefore wch can be demonstrated of centripetal forces of bodies revolving in ci[...]

demonstrated of these centrifugal forces in their mom[...]

This was first done by Mr Huggens, & shall be [...]
mentioned in ye present course, because it gives us ye mea[...]
of mustering by experiments ye doctrine of central forces in free motions, and at ye same time gives us our only argument against a system of philosophy, which was published by Descartes, & an explanation of ye planetary motions.

1. $F \div \frac{D}{P^2}$

2. If $P \div p$ then $F : Q \div D : d$

3. If $P^2 : p^2 \div D : d$ ye $\frac{D}{P^2} : \frac{d}{p^2} \div \frac{D}{D} : \frac{d}{d} = 1 : 1$ and therefore $F \div Q$

4. If $P : p \div d : D$ then $P^2 : p^2 \div d^2 : D^2$ and $\frac{D}{P^2} : \frac{d}{p^2} \div \frac{D}{d^2} : \frac{d}{D^2} \div d : D$ therefore $F : Q \div d : D$.

5. If $P^2 : p^2 \div D^3 : d^3$, then $F : Q \div \frac{D}{P^2} : \frac{d}{p^2} \div \frac{D}{D^3} : \frac{d}{d^3} \div \frac{1}{D^2} : \frac{1}{d^2}$.

6. The centrifugal force is proportional to ye square of ye angular velocity multiplied by ye distance. For ye angular velocity, or ye angle described in any given time, is inversely as ye time of describing ye whole circumference that is calling ye angular velocity A / $A \div \frac{1}{P}$. Therefore $\frac{D}{P^2} \div D \times A^2$, and

[...] A is much used.

[...] centrifugal force is to ye weight of ye body as ye height thro' w ye body must fall by ye uniform action of its weight, in order to aquire ye velocity of ye revolution, is to half the radius of ye circle in w it revolves.

For it has been shewn ye if a body be retained in a circular orbit by a centripetal force, ye uniform action of ye force would communicate ye velocity of ye revolution by a fall thro' ½ of ye radius.

The uniform action of the gravity is supposed to communicate ye same velocity by a fall thro a certain height **H**. It was also formerly shewn ye if two powers **F** & **G**, acting uniformerly, communicate equal velocities by different falls, the falls are inversely proportional to the forces therefore $F : G \div H : \frac{1}{2}R$. But what was formerly demonstrated of centripetal forces is also true of centrifugal forces of bodies revolving in circles. Therefore the corollary is true.

Hence we aquire ye method of obtaining ye absolute quantity of ye centrifugal force of a body in any point of any curve. For it may be considered as revolving for a moment in ye nascent arch of ye equicurve

universally, ye centrifugal force of a body in any

motion in any curve is to the weight of ye body as H is to

radius of the equicurve circle.

Nature exhibits many instances of Centrifugal force. A stone swinging round in a sling is a well known instance, & ye strain upon ye string is ye effect and measure of ye centrifugal force. The fanners of a bolting mill, the fanners for winnowing grain & ye circular ventilators, are machins acting on this principle. The experiment w I formerly mentioned for ascertaining ye difference between absolute & relative motion is another instance. In consiquence of ye centrifugal force arising from ye Earths motion round her axis, she has deviated from ye sperical figure, & acquired a sort of turnip shape. From ye same cause gravity is diminished as we approach the Equator, and did she revolve about her axis about 17 times

she does bodies at ye Equator would have no

all. Jupiter is flattend for ye same cause.

Of the Equilebrium of central Forces.

If two bodies are connected together & made to revolve round a point, from w their distances are inversely as their quantities of matter, their centrifugal forces will ballance each other, and ye point will rest.

Before quitting this subject we must not neglect a curious observations w has appeared a great difficulty in ye science of motion, & has led one of ye first Geometers of Europe into mistakes concerning ye revolution of bodies in constrained curvelineal paths.

It has been demonstrated that ye revolving body suffers no loss of velocity. It has also been demonstrated that it exerts a determined pressure on ye canal, w must be ballanced by a contrary resistance

[We] have considered only those changes wch were [produ]ced by a constrained path on ye motion of a body [w]ith its own inertia. But ye most important phaeno-mina occur in those motions which are ye production of external forces.

All ye important questions on this subject may be reduced to two.

1. to find ye velocity in any part of ye path. 2. to find the time of arriving to any given point.

1. Let a body A [Fig. 25] be impelled by a force directed to S along a line ATR, all lying in a plane passing thro' ye line AS. About the centre S discribe ye circle TD. Then knowing ye distance AS from wch ye body begins to move, & ye law of the centripetal, we can find ye velocity acquired by ye rectilineal motion at ye point D; and it may be demonstrated in the same manner as in the case of free motions, that the velocity at T is equal to ye velocity at D. 2. Divide AS into innumerable equal parts, of wch let Dδ be one – draw A2 perpendicular to AS describe ye arch δT. Now Dδ being given Tt is also given, for Dδ : Tt = Rad : Secant of ∠ STt. Since ye velocities at D & T are equal the ti[me]

of describing Tr is given, being equal to $\frac{Tr}{v}$, [...]
constant & therefore equal to 1 the time of descr[...]
$\frac{oc^t \angle STr}{v}$. Make DN perpendicular to AS and pro[por-]
tional to this quantity, and let PNO be y^e curve which
passes thro' the extremeties of all y^e lines such as DN. The[n]
the times of describing AT will always be proportional to
y^e area $\angle$PNO. For it is plain that $D\delta \times DN$ or the area
$D\delta n N$ is the fluxion of the time, & y^e area $\angle$PNDA will
be the whole time.

In motions of this kind it is evident that
the path sustains a twofold pressure one arising from
the centrepugal force, w^h was proportional to $\frac{v^2}{r}$, and
another arising from the resolution of y^e central force
OT into TK w^h promotes the motion in y^e curve, and
KO, w^h increases y^e pressure on the path.

In these disquisitions concerning the constrained motions of bodies along determined paths, w they cannot quit, we have conceived them confined to the paths by its solidity & immobility.

But there are other, and more common ways in w these constrained motions along curvelineal paths are produced. Suppose, for instance, that the body A (Fig. 26) to be fastened to the thread AE, w is applied to the curve GKF, by y evolution of w the curve GAH is produced. Suppose that the body is acted on by an accelerating force directed to a point S, so situated that a line drawn from it to any point of the curve lies wholly without it. It is plain that the body will be made in this curve GAH. For at any point A, the accelerating force, w may be represented by any line AB may be resolved in AC directed along the tangent, & AD directed ... momentary radius FA. By means of the force AC it is impelled along the curve, and by the force AD it stretches the thread, and is opposed by an equal & contrary force AE, exerted by the cohesion of the thread, and by its means propagated to the momentary centre F. The evolute, therefore, performs in the present case the same effect w was performed by the solidity of the path in the former cases. Now however various these curves may be, the motions in them are precisely the same as in the nascent arches of the circles, having lines such as AF for their momentary Radius & points such as F for their momentary centres. If the thread is fastened to a fixed point, & does not unfold from any curve, the curve described by the body A is a circle, as in the case of a common pendulum. There is yet another method which deserves our particular considerations, as it is the foundation of all mechanics.

Suppose a body A (Fig. 27) connected with a fixed point P by means of an inflexible line AP, which is at liberty to move round P. The body is by this means constrained to move in a circular path, having in every point the direction of the tangent

The motion of such a body is to be determ[illegible]
there. Let the accelerating force act in the direc[illegible]
represented by AF. Drawing FB perpendicular to [illegible]
along y motion is directed and FC perpendicular to AF
plain that the part FC acts only in pressing the line dir[illegible]
on the point P, & is opposed by the resistance of y path in the
direction AE. The force AB impells it along the curve. and if AF
represents the space thro' w the accelerating force would have
carried the body if moving freely, AB will represent the space alon[g]
w it would impell the body in the direction of its constrained motion.
Mean while the point P must be sustained by a force equal
to AE or AC. Draw PD perpendicular to AF; then on account
of the similarity of the triangles FAB, APD. we have this
analogy FA : AB = PA : PD. Therefore if PA represents y abso
-lute force of F, PD will represent its effort on the body A, by w
it impells it along the arch of the circle, or produces y rotatory
motion. Therefore its absolute quantity is to this effort as the
distance of the point of application from the fixed point to the
perpendicular distance of its direction from the same point.
Let the same force act in another direction Af, and drawing as be-
fore fb, fc Pd, its effort will be represented by Pd.
Hence the efforts of a force applied to the body are as the per
-pendiculars let fall from the point P on these directions.

P is called the centre of rotation or motion ~

Let any other force Q

... therefore be ascribed to the other qualities of matter, which therefore must be considered by us as powers. Our only notion of these powers is that they produce changes of motion; this is the general point of view in which we are to consider them & by which they are all brought into one class, however they may be distinguished by circumstances peculiar to each. Thus the actions of animals, the force of gravity, the impulse of wind, water, or solid bodies all agree in producing motion; and are therefore considered by us as powers. But their nature is different in each: But what notion have we of this difference, but the difference in motion which they produce. We must, therefore, have some principles previously established with respect to motion before we can assert the existance or influence of any moving power. And we must perceive the circumstances in which all powers agree in producing or modifying motion; before we can ascertain those in which they differ, and serve to charecterize each particular class. The first object of our attention, therefore, must be the general laws of motion and moving powers. These I shall consider under the general expression Dynamics ——

In this part of our subject I shall consider a particle of

matter as agitated by moving powers every general
case in which it can be found, and thus establish a set
of general laws, by which we are afterwards to proceed
in investigating the nature of powers whose exertions
we observe in the different classes of natural Phenom=
=ena; I. E, in investigating the nature of the subordinate
properties which distinguisheth the different classes
or situations of national bodies. This disquisition
must be looked on as the abstract and difficult
part of the course. For as I shall dwell entirely
on general ideas, & abstract from every distinction
among bodies, I cannot give instances where these diff=
erences unavoidably take place. But it will be attend=
with this advantage that the subsequent parts of the
course acquire a connection which you will hardly
find in system of Natural Philosophy, and the
investigation will be, upon the whole more perspicuous
and shorter. For in the usual manner of treating
this science each branch of it had a set of first prin
ciples belonging to itself which it was unnecessary
to investigate, because not dependant on more

general principles. I flatter myself, therefore that
you will find the close attention which you will be
obliged to give to the first part of our course amply
repaid by the facility you will afterwards acquire in following
out my explanation of the Phenomina of nature.
Having thoroughly established these general principles
of Dynamics, I shall in the next place proceed to the
philosophical history of the principle classes of natural
Bodies. In ~~different~~ classing natural bodies different arrang
ements suit different purposes. The nomenclator or as
he is commonly called the natural historian chooses
for his principles of arrangement the resemblance
in external forms. The Chymist chooses the resemblance
in their relations to heat and mixture. The Physician
their resemblances in their mode of affecting the
annimal constitutions. I take for the principle
of my arrangement the resemblance in their
properties with respect to motion.
Thus as all bodies on which we can make experi
ments are heavy, I, E, when unsupported falls or begin
the application of our general doctrine of Dynamics with the
Phenomina ~~from which the truth or reality of the~~

of gravitating bodies, shewing the Phenomina from which the truth or reality of general property is inferred, and then explaining all the subordinate Phenomina. This will include Physical Astronomy, the theory of gunnery, the motion of pendulums, and vulgar Mechanics. ______ The next mechanical property of matter, in point of generality, is that by which the parts of matter remain attached to each other and resist the efforts to seperate them. This is not so general as the other, for we shall find that it is not competent to all matter, nor to the same matter in all situations, being susceptible of two principle modifications called solidity and fluidity, and each of these being susceptible of other modifications called compressibility and incompressibility, the compressible bodies of each class showing a remarkable property called Elasticity, which produces very great variety in their properties with respect to motion.

The facts which occur when considering the Phenomina of heavy bodies will point out to us an analogy by which we are enabled to make some probable conjectures with respect to the nature of this power of Cohesion, and its

different modifications. After discussing these I shall proceed to explain the mechanical Phenomina consequent on the combination of the general principles of Dynamics with the new properties of gravity and cohision.

This will lead to the consideration of the strenghts of bodies, the communication of motion by impulse, or Collision, the laws of Hydrostatics, Hydraulics, and Pneumatics, Acoustics Resistance of media, and Friction. Having thus exhausted the Phenomina which depend on the most general properties of matter, I shall in the next place proceed to consider such as arise from its more particular affections. Here the most remarkable is that which certain bodies acquire in various ways of attracting and repelling other bodies. I mean Electricity, of which I shall attempt a mechanical theory, from analogy. The ~~the not~~ mechanical Phenomina of Electricity are generally accompanied by some others very wonderful and important. These tho not mechanical, shall be considered, in hopes that in future time their connection with the mechanical properties may be investigated — Analogous to this are the Phenomina of magnetism, of which I shall also deliver a mechan ical theory from analogy, which perfectly concides with every Phenomina.

I shall in this manner have ex=hausted all the Phenomina of the sensible motions and actions of bodies upon each other. In the prosecution of the study we shall have seen such varacity of moving powers, that we shall find great analogies between these Phenomina and those which take place in the sensable motions & actions of matter — I shall therefore take an opportunity of informing you of the attempts made by ingenious men to explain the Phenomina of heat, mixture, and the growth of plants and animals, and give you an account of the best suppor=ted opinion concerning electrical attractions.

This remains a very numerous and important collect=ion of Phenomina, which I dare affirm, to be causes of moving, bodies and therefore properly comprehended in our plan — I mean the Phenomina of light — But it has also been considered in a course of this kind; and I hope that I shall suggest reasons which will be sufficent to induce you to believe that it may be reduced under the Phenomina of moving bodies —

Such is the arrangement that I propose to follow in the course of Lectures on which we are now about to ~~follow~~ enter — During our progress there will occur many applications to the arts of life which would take up to much of our time to consider fully, but which are of too great importance to be passed altogether. I shall make a collection of them and connecting them in the best manner I can make them the subject of a particular course —

Introduction

Of Space, Time, and Motion

1st While a body moves time flows, and the body passes thro' the contiguous parts of space during the successive parts of time. In order therefore to have a distinct notion of the motion of bodies, it is necessary to have clear conceptions of space and time.

2d Space in general is an abstract term expressing a simple idea; and therefore incapable of any definition; and all we can do is to reflect on the manner in which we form this idea

3d Two points give us the notion of distance this is the simplest notion of space and the distance we call a line Extension of one dimension or measure, because it has it has no measure but lenght.

4th Three points may give us ~~another~~ the idea of another kind of space if one of them does not ly between the other two, or in the straight line joining them. This kind of space is a surface and has two demensions, for it may be measured both in lenghts and breadth.

5th Four points may give us the Idea of another kind of space, if none of them lys in the line or surface joining the others — a solid, having three dimensions, for it may be measured in length, breadth, and thickness.

6 A point seperates the parts of lines but makes no ~~point~~ part of either; it is only the boundary where one ends and the other begins — A line in like manner is the separation of two contiguous surfaces, but is not a part of either — and a surface separates two contiguous solids but makes no part of either

7 Space is unbounded or infinite What is beyond any assigned bounds? Space — it is infinite, for no space is so great but we can conceive a contiguous space equally great and therefore there is a space which holds both

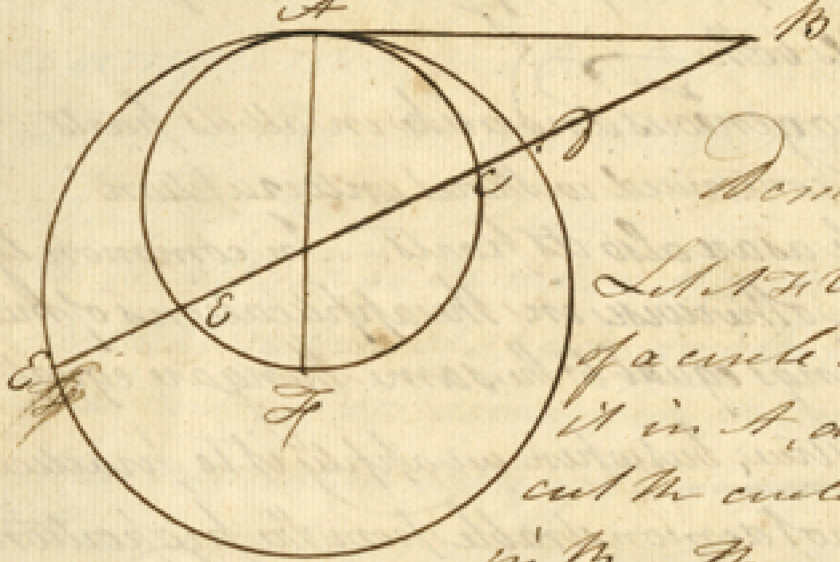
8 It is perfectly homogeneous and alike in all its parts

9 It is contiguous or continued without interruption

10 It is immoveable as are also its parts — In common life we act as if it were otherwise in the applications of measures saying that two things equal to the same thing are equal the proportion is true, but when we apply it to practice, our practice is not demonstrable from the proportion. when I measure two lines by means of a third, which I successively apply to each, I do not apply the same numerical measure or part of space, but a different.

11 Space is infinitely divisable. For as space is continued and uninterrupted, and as no point in any part of a line, it follows that between every two points a line is interposed a 3d point may be placed between them without touching either

for it may be placed either equally or unequally between them
if it is equally it cant touch one without touching both,
and thus there will be but one point instead of 3 or 2
if it is placed unequally and be supposed to touch one,
it may be brought farther from it without touching
the other.



Demonstration

Let AF be the Diameter of a circle and AB touching it in A, and any line BCE cut the circle in C. E, and AB in B. — Then $AB^2 = EB \times BC$

Describe now another circle cutting BE in E and D — then $AB^2 = EB \times BD$. or $BD = \frac{AB^2}{EB}$, the larger that EB is the smaller will BD be, but there is no bounds to the enlargement of EB, & therefore none to the diminution of BD.

12 Space is the place of bodies

13 What is now said refers only to space in the abstract, but our real perceptions of space are very different & imperfect, the space of itself could not have been conceived by us, and it is only by means of bodies that we perceive it, and perceive it so far as it is the place of bodies, and while bodies do not only simply change their distances from each other and from us, we imagine the part of Space to be the same. When I return to this room to morrow, I think I came to the same place, even altho reflection tells me that the place I am in now will be at an immense distance from that part of space in which I shall be tomorrow when in this Room

14 Should the whole universe be moved from its place, we could not perceive any change of place, and should imagine ourselves, in the same part of space. Nay should it all contract or swell in any proportion the change could not be ~~[illegible]~~ sensible provided only that the laws of attraction and repulsion were changed accordingly

15 On the contrary should a change happen in the positions of objects, there would result a change in our Ideas, but this change will be the same, whether

the change of position happens to ourselves or to external objects. The idea is perfectly the same whether the sun turns round the Earth or Earth round the sun; so different is sensible place from real and relative place from absolute.

16 It is this sensible space which we call place. This also refers to contiguous bodies and therefore were there but one body in the universe which could assign it no place.
The true place of a body is that part of absolute space which it fills. Its sensible place is its situation not place which it has among other bodies. Its sensible place therefore may vary while its real place remains the same, namely by the change of the situations of surrounding bodies. The sun nearly retains his real place, but his sensible place continually varies, and is not the same to any two spectators. We can never therefore assign the real place of any thing unless there is some body which never changes its real place; and to which we may refer all the others. These sensible measures of space may well enough answer the purposes of common life, but they are unfit for the purposes of Philosophy, in which we must abstract from appearances & consider realities.

Of Time

17 Much of what has been said of space may be affirmed of time.
It is uncapable of definition. We got our notions of it by the successions of our thoughts or perceptions.
A beat of my watch gives me no idea of time; but two beats convey the idea of duration interposed between them it is evident that a 3d beat will give me the idea of another duration. We can feel an equality in the intervals, but have no distinct idea of their proportion. We follow the same method as in space we take what we think a common measure the intervals between the beats of a watch or the like. But whether we can or not ascertain the perfect equality of two portions of duration, we know that two portions may be equal, and therefore double of one of them, that there may be a portion which is triple another &c. Thus we know that time is a proper quantity altho we may not be able to measure it distinctly.

18 The beats of my watch are not parts of time, but only the

bondaries of these portions of time which ly betwan them; and are called instants; while the very smal parts of duration which flow between them are called moments

19. In all these considerations there is such an analogy between Space and time that we cannot talk of time without borrowing language from space. Thus the terms of space; Interval, long, short, near, before, after belong properly to space but are applied, in our language at least, to time The Russian language has a set of words appropriated to each ποpa; π

20. Hence points are mentioned in time if it is represented by a line. one remarkable difference the quantity observable in space may be measured in three different ways that obsevable in time only in one way

21. As no two points of space can be coincident, so no two instants of Time can be simultanious or eaistant. The indentity or numerical difference of two points of space is that they are at distance, that of two instants of time that they are successive

22. Hence it follows that Time may be devided without end no interval so short but an event may happen after its beginning but before its ending

23. Time like space must be unbounded, that is eternal, uninterupted, homogenious, immoveable or unchangable in the order of its parts

24. Time is the Ulra of every event. All things are placed in space in the order of situation, and are placed in time in the order of succession.

25. All that is here delivered is true only of Time in the most strict and obsract sense. But we cannot perceive this time. As all its parts are perfectly alike we cannot distinguish them each other or tell which of them any event happened. Had the whole world eaisted many millions of years sooner or later we could not know it provided only that the same intervals were interposed between the events which we percive. Had the duration of the universe been lengthned or shortned in any

proportion it does not appear that we could know it; for we have no information by which we can learn the change or immutability of the succession of our ideas, and therefore we are always disposed to refer the changes to changes in the interval. But reason, that is observation makes us correct this hasty judgement. A man intent on study thinks the time short but trusts his Watch

26. Thus then the real time is unknown, but there is a sensible time like a sensible space, bearing a relation to some trains of events. Thus we say that the fire of London happened in 1666 altho this by no means points out the real Æra of that event.

And time is measured by successions of events. beats of a Clock mark seconds, successive sun settings mark days, successive changes of the moon mark months, and successive changes of the seasons mark years. These are assumed equal altho their differences amount to a considerable quantity which Astronomers have discovered and called the equation, and by which they endeavour to point out the difference between true time and the sensible measures of it, which we commonly use in its stead

27. Our sensible time may therefore be very different from the real. A Person who sails round the Earth Eastward and returns to the same place will have reckoned two days more time than the man who made a similar voyage westward.

[...]omena which they occasion among bodies in relative mo-
[...]on are sufficient authorities for judging of their nature-
Nay we may go farther- There are many circumstances in na-
-ture from wch we can with certainty and accuracy discover absolute
motion, and assign its quantity. These are the causes which
distinguish them in nature, namely the powers which pro-
duce motion. True motion can neither be produced nor chan-
-ged with force being impressed on the same bodie. But a rel-
-ative motion in a body may be produced by only changing
the situation of the neighbouring bodies- again true mo-
tion is always changed by any force impressed but a relative
motion is not so changed for if the same forces are impressed
on the bodies to which the referince is made their situations
remain ye same, and no change happens in their relative
motions. Thus if two ships are sailing from or toward each
other on any point of the compass, in stile water, and both of
them come at one time to be carried away by the same curr-
-ent their relative motions will not be changed-
There are also effects by which we can at all times distin-
-guish real from relative motions. The chief of these is
that tendency which bodies, moveing in a circle, have
of receding from the Centre. If the circular motions in on-
ly apparent this tendency is nothing, but in absolute
motions is greater or less according to ye quantity of abso-
-lute motion-

Sir I N illustrates this by a beautiful and se[illegible]
-periment. Let a bucket of water be hanged by a [illegible]
and twisted about till the string is much twisted. When
every thing is still let it now be suddenly turn'd round
the other way. The water will not immediately aquire
the motion of the bucket, and therefore it has a relative
motion contrary to that of the bucket. But by degrees fri-
-tion of the sides will communicate motion to the water,
and therefore its motion relative to the bucket will con-
-tinually decrease, and if the motion is long continued the
water will at last move as fast as the bucket, and there-
-fore will be relatively at rest in it.

But it will be found that ye surface of the water will be
level at first, and will gradually heap up at ye sides, the
water receding from the Centre as the real motion incre-
-ses and the relative diminishes. The true circular motion
can by no means be ascertained by the relative motion but it
may be ascertained by means of this Centrifugal force.
The relative quantities therefore are not the quantities
whose names they bear but only sensible measures, true
or false, which the vulgar mistake for the quantities them-
-selves. But we see that we can distinguish the absolute
from the relative, partly by ye apparent motions,

[s]uch are the differences of ye real ones, and partly by the
forces wc are the causes or effects of the real ones.
Thus if two spheres connected by a thread revolve round
that point where they are in equilibrio, the tension of the
thread would show their endeavours to recede from the
Centre, and therefore show us the quantity of their cir-
-cular motion.

If now we impress on both bodies in opposite directions
equal forces if these conspire with their Circular motion
it will increase the tension of the thread, but if it coun-
-teracts their Circular motion it will diminish the
tension. Thus by trying all sides of the bodies, we
can find what side is that which upon being struck
produces the greatest increment of the tension. This
will doubtless be the posterior side. Thus the direction of
the Circular motion is found out.

From such an observation as this we can discover all the
real motions which obtain in the observed relative motions.
For we can compare the discovered motion of our spheres
with the apparent motion of the constellations and
thus discover whether they have any motion and what
it is. knowing this it is easy to determine all the
other motions in the universe.

This is to constitute our future business

Having thus established some general notions of space Time, and motion, we come in the next place to consider the more general properties of motion, deduced from a consideration of the Circumstances which concur in forming the Idea. All that we can observe in motion is the space passed over, and the time imployed. Motion has a beginning and end in space; it has also a beginning and an end in time.

The portion of time elapsed is the duration, or time of the motion

The portion of space passed over is the path of the motion. It is suceptible of three varieties, in position, form and length.

The position of the path along wc motion is performed is called the ~~duration~~ direction of that motion

If the direction of the motion never changes, the path is a straight line, and the motion is called rectilineal but a change in the direction makes a change in the path, which therefore ceases to be a straight line, and may have any other assignable form, for any figure may be discribed by motion. In every point of the path the motion has a certain direction, according to which it is describing the part of the path passing

of that point. When the path is a curve, in which case he motion is termed curvelineal, the direction is the position of the tangent to the curve in that point. These varieties in the path of motion determine its kind. The other variety in the length of the path determine the quantity of motion, for it determines the quantity of translation of the moving point.

A motion is similar to another when its duration, path, and direction are similar. it is dissimilar when any of these are dissimilar. If the time is longer, it is called slower, if shorter, faster. If the path is longer, it is called swifter, if shorter, it is called slower.

The terms swift and slow, therefore, do not depend on the space or time alone, but on both jointly. The quality by which motions differ in this combination of ye Ideas of space & time is called velocity in the language of Natural Philosophy. It is there an improper term because it alludes to one form of this combination only namely when the Idea of a great portion of space is combined with that of a small portion of time.

We have in the English language a very conveneint term expressing such a combenation or expressing how much of one thing corresponds to a certain quantity of another. We say that the money was lent at the rate of 5 pr Cent. that a field produced at the rate of 40

bushels an Acre, and we say that a ship sailed at the rate
of nine miles an hour —
The term rate in all these cases has the same meaning
It does not express a proportion, for this can obtain only
among homogeneous quantities — But rate always means
quantities of different kinds. Even when we say five p.r
Cent. we mean different kinds of quantity, namely in-
-terest and principal, and in general, the term rate expr-
-esses how much of one thing corresponds to a determined
unity of another —
It is in this sense that we are to use the term velocity, not
as expressing any proportion subsisting between space
and time, but only the proportions which subsist a-
-mong the different spaces corresponding to some deter-
-mined unity of ~~space~~ time — The first however, is the most usefull.
We seldom say that a man wakes at the rate of two hours
to the league, but we say that he waks at the rate of half a league
an hour —
When the Natural Philosopher speaks absolutely of ve-
-locities he speaks of the space corresponding to some uni-
-ty of time, usually a second — But when he speaks of them
comparatively, he seldom assigns them absolute mea-
-sures. Then when he says that two velocities are as 10
to 11 he says nothing from which we can infer the real

quantity of either; and the expression only ascertains the
proportion of the spaces corresponding to any indetermin-
-ed unit of time, and the numbers themselves express the
proportions which each space bears to some one space con-
-sidered as unit or that each velocity bears to some one velo-
-city considered as unit, so that the proportion of ~~these pro-~~
~~portions~~ — 10 to 11 is the proportion of these proportions —
It must be here observed that this Idea of the velocity is
only an Idea of the measure of that quality of motion
by which a certain quantity of space comes to correspond
to a certain quantity of time, and that the term velocity
is only an expression for the measure of this quality.
The quality of itself is an abstract Idea which we can-
-not define, but only discribe by its measure. When a
body falls, it moves faster and faster every moment,
and we say that its velocity is contenually augmen-
-ting. Now in any one instant of time it does not move
over any space but is only moving over it, and it is im-
-possible from the fact to assign any rate of the space
and time; for let us take any time however small, its mo-
-tion at the end of that time is swifter that at the begin-
-ning, yet we pretend to assign its different velocities at every
point of its fall. The measures which we assign in these

cases are the measures of the velocities when they
=posed to remain constant for some length of time or
space. Here then the velocity means a quality of motion
which we consider as a quantity, not directly, but by
means of some measure with which it is invariably con-
=nected; and in order to obtain such a means we are ob-
=liged in imagination to suppose that the quality or ve-
=locity remains for some time unchanged. But it is
evident that in cases where it is continually changing
we shall always err from the true proportions. But
the smaller the time, the nearer will our measures
approach to the true proportions, and therefore when
the time vanishes the proportions of our measures will
be the same with those of the quality—

This is the foundation of the maxim which we shall
be continually employing, that during an infi-
=nitly small part of time, or along an infinitly small
part of space, the velocity is constant—

Uniform motion is that when the velocity is constant.

Accelerated motion is that in which the velocity increa-
=ses—

Retarded motion is that where the velocity is demin-
=ished—

uniformly accelerated, or retarded, motion is that in which
the velocity receives equal increments, or decrements, in
equal times—

I may just observe that it is in the respect only of the time
that the motion of Natural Philosopher differs from that
of the Geometer— In elementary Geometry figures are con-
=sidered as produced by motion, but without any regard to
the time, and it was reserved -- for Sir I. N. create a new
and sublime spaces of Geometry by introducing this Idea
into the motions considered—

Since in all varied motions the circumstances are contin-
=ually changing, in order to have accurate measures of
them we are obliged to suppose them uniform for a small
moment of time, and from the properties of uniform
motion we determine the momentary properties of any
motion however varied—

It is therefore necessary to consider the chief properties
of uniform motions. They may be reduced to three propos-
=itions—

Prosposition 1—

In uniform motions the spaces are proportional to the
velocities, the times being the same—

This follows from our conception of velocities, which
mean nothing but the measures of a certain quality

Now this measure is the space run thro' uniform[ly in] a given time— In saying therefore, that the spaces are proportional to the velocities, we either mean that spaces are proportional to themselves, or that the measures are proportional to what they exactly measure, which must be true as to any judgement we can form, for this must be taking from the measures which we suppose invaribly connected with it—

Proposition 2—

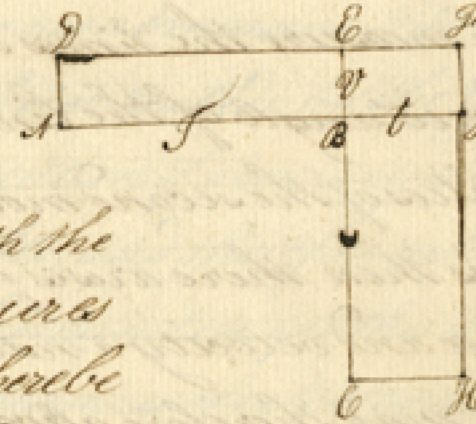
In an uniform motion the spaces are proportional to the times, the velocities being equal— With respect to our judgements in this mater proposition may be demonstrated in the same mañer. For the spaces described an uniform motion are assumed by us as the only measures of the times elapsed, and must therefore be proportional to them—

The proportions of the parts of time are of themselves unknown, and therefore it cannot be from any perception of an aguement that we afferm the proportions of spaces to be the same. It must be only in consequence of the measure which we assume, and our assurance of an invariable connection subsisting between time and its measure. This is not the place for adducing our reasons for

[be]lief, and it is sufficient at present that we have no other measure of the times but the spaces described by uniform motions, which must therefore be proportional to them—

Proposition 3—

In uniform motions the spaces are proportional to the rectangles of lines measuring the velocities and times— This is a deduction from the preceding purely Geometrical—



Let the two motions be performed during the times whose measures are A B, B G with the velocities whose measures are E B, B C, and let there be a 3d motion performed during the time whose measure is B G, with the velocity whose measure is E B—

Then since the first and third motions are performed with one velocity measured by E B, the spaces will (prop. 2) be as the times measured by A B, B G, and therefore as their measures A B, B G, therefore they will as the rectangles A E, B F—

Again since the third and second motions are performed

in one time measured by CG, the spaces will be proportional [torn]
to EB, BC, or to the rectangles BF, CG —

Now the space described in the first motion is to that described in the third motion as the rectangle AE, to the rectangle BF, and the space described in the third is to that described in the second as the rectangle BF to the rectangle CG, Therefore the space in the first motion is to the space in the second motion as AE : CD or as the rectangles of the lines which measure the times and velocity of the first motion to the rectangle of the lines which measure the times and velocities of the second motion —

Thus then there arises from the nature of our measures of time and velocity a natural measure of the space, a space may therefore always be represented by a rectangle of an Area proportional to the space, but its sides indetermined, and now haveing determined any one of them by a knowledge either of the time or velocity, the other comes to be determined by that problem of elementary Geometry where we are taught to apply to a straight line a rectangle equal to a given rectangle — The other side of the rectangle formed by this Problem will be the measure of the velocity or the time, according as the given side was the measure the time or the velocity —

This is the meaning of y^e Latin expressions spatium est proportianale velocitati ductae in tempus, and velocitas est proportionalis spatio applecato ad tempus, and tempus est proportionale spatio applecato ad velocitatem.

As a product of numbers is anologous to a rectangle of lines, it follows that the spaces are in the proportion of the products of numbers which measure the times & velocities. This is a much more convenient measure than the other, and though some writers choose to abide by the Geometrical measure, from an affectation of accuracy in quantities which are incommensurable, and therefore can't have their proportions represented by numbers, yet this is an unjustifeable necety. For our measures of velocity and time refer to some quantity of each reckoned as unit in every case where we give them absolute measures, and therefore we can represent their proportions by numbers without relinquishing the Geometrical expressions when it is more convenient —

Since Division in numbers corresponds to applying a rectangle to a line representing the Divisor, and since the Quotient will in this case be represented by the other side of the rectangle it follows that when a space is given and we [torn]

want to find the velocity, we need only divide the nu
representing the space by that representing the time an
the Quotient will be the number representing the velocity
Hence the Arithmetical expression, the spaces are propor-
-tional to ye times multiplied by the velocities, and the times
are proportional to the spaces divided by the velocities,
and the velocities are proportional to the spaces divided by
the times—
When two rectangles are equal to ye bases are reciprocally
proportional to the heights, when two dividends are equal
the Quotients are reciprocally proportional to the divisors
The greater the height the shorter will be ye base the
greater the divisor the smaller will be the Quotient—
When a side of two rectangles is equal, the other side is direct-
-ly proportional to the rectangle— when two divisors are e-
-qual the Quotients are derectly proportional to the dividends
Hence the truth of these proportions—
These spaces are derectly proportional to times and velocities
The spaces are derectly as the rectangles or products of ye times
and velocities—
The spaces are in the ratio compounded of the ratios of the t

velocities—
The velocities are in the derect rates of spaces and inverse rat
of the times—
the Times are derectly as the spaces and inversly as the velocities
All these propositions are more conveniently represented by
general symbols or formulas

1. $S \div VT$,	4. $S \div T$
2. $V \div \frac{S}{T}$	5. $S \div V$
3. $T \div \frac{S}{V}$	6. $T \div \frac{1}{V}$
	7. $V \div \frac{1}{T}$

I may here make one observation wc will be of frequent use.
If any effect changes at the same rate with each of
many different causes, the effect will be derectly as the
product of all the causes which increase it while they
increase, and inversly as the product of all those which
deminish it while they increase—
It is demonstrated in the very same way in every instance
as in the present, and therefore I shall without hesita-
-tion apply it as I go on— I shall use it at present in es-
-tablishing another expression which is to be of frequent use.
As the change of place is measured by the path of the motio

and this depends on the velocity, the quantity of m[illegible]
of any point will increase with its velocity. But the [illegible]
-solute quantity of motion which may be the subject of
our consideration will always with the number of mov
-ing points. When therefore a body is in motion, and of
consequence each particle in motion, the absolute quan
-tity of motion must be made up of the sum of all the
particular quantities of motion. It may therefore be
represented by a number. If all the particles have one
velocity, this number will be the product of the number
of particles multiplied by the number representing
the velocities; or it will be by a rectangle one of whose
sides is analogous the velocity and the other to the num
ber of particles—

Hence if Q represents the quantity of motion, M the
quantity of matter, & V the velocity of the body we have
the following expression—

$$Q = MV;\ M = \frac{Q}{V},\ V = \frac{Q}{M}.$$

[illegible]d on by a smooth current: while another ship in sight
[illegible]lmed in still water, each appears to the other to move—their
[illegible]stance or situation, or both, are continually changing, but
the appearance is the same in both ships, and it does not
appear whether this effect is produced by a motion in the one or
in the other, and by spectators in each the motion ascribed to
the other—

In like manner that motion which is the cause of rising
and setting of the heavenly bodies is ascribed by a spectator on
the Earth to a motion in them from East to west; but a motion
of the Earth from west to east would produce the same appearance
viz a continual change of the situation of the heavenly bo
-dies with respect to the Earth— Which of these is the cause
has been the subject of long and difficult investigation—

Thus tho' these necessary effects of real motions are continual
-ly approaching to us, being the difference of real motions,
the real motions themselves are very difficult to be discove
-red. For when we are carried along by a motion of which we
are insensible, we imagine that we are at rest, and assum
-ing this false principle in our hasty & imperceptible reason
-ings about the apparent effects of real motion, we mistake
our false conclusions for real perceptions, and we call these
errors of judgment deceptions of sence—

It would appear therefore that we must remain ignorant
of the absolute and real motions of bodies, and consequen[illegible]

-ly ignorant of the powers of nature which pr[illegible]
regulate motion. For since we have no notion of th[illegible]
powers but by their effects, and since their effects are ye
changes of motion, it would appear that the real causes
must be as different from the apparent causes as the real
motions are from the apparent, and that we must there-
-fore remain ignorant of them. But this is not the case—
Whatever are the absolute motions of two bodies, their rel-
-ative motions are the differences of their absolute motions—
suppose a man should walke from the stem to the stern
of a ship while she sails a mile, his relative motion is a ship
length, this is also the difference between the absolute moti
of the ship and man— This relative motion would have
been the same and the difference of the absolute motions
equal to it altho' the absolute motion had been any how
different— Now it will be found that the action of bodies on each
other in ye way of impulse does not depend on their absolute motions
but the differences of these motions. The sensible actions there-
-fore of bodies on each other in respect of impulse are sufficient
for giving us a just notion of the powers of impelling bodies. It
will also be shown that the powers of gravity, electricity magne-
-tism and the like, are not affected by any motions, and therefore

...ye at midsummer noon, I discharge a 24 pounder, from some part of the earth wc has no latitude, due East, & suppose yt it were possible to make ye bullet fly with about 150 times ye velocity yt our greatest charge has ever been able to give. The consequence will be yt it will set out wt an inconceivable swiftness, wc will gradually diminish, & at last ye bullet, after having flown a few miles to ye East. will touch ye ground, where it will rest. Here is a passage from a most evident motion to rest. But ye fact is just ye inverse. The bullet ye moment it is out of the piece is nearly at rest. But ye air is rushing to ye westward wt a velocity of near 100,000 feet in a second, & by its producing resistance gradually communicates its motion to ye bullet, till at last, ye bullet is moved by ye Earth, & acquires ye whole of its motion in its orbit round ye sun. A person who remains for twelve hours in ye place where ye gun was discharged has acquired an addition to his motion of about 2500 feet in a second, three times swifter than that of 24 pounder when employed to batter in breach, yet of ye change he is quite insensible.

What then remains to be done in order to settle ye important point, the foundation of every enquirey of Nat. philosophy. It is not so difficult as we would imagine altho' we cannot observe ye absolute motions of bodies, yet their relative motion will enable us to judge of ye laws of nature wc obtain in ye absolute motion.

Now daily & uncontradicted experiment shews us y ...
y same force at mid summer noon to move a body Eastward ...
westward. The one is a deminution & y other an equal augmentatio
of absolute motion. A body therefore shews no greater tendency
to diminish than to increase motion.

Farther; y relative motions of bodies are y differences of their absolute motions. canges of relative motions are therefore equal canges of y differences of absolute motions. If two bodies persevere in their absolute motions they persevere in y differences of these absolute motions, & therefore in their relative motions. & y differences of their relative motions; on y contrary if two bodies persevere in their difference of relative motions, they persevere in their difference of absolute motions. and in their absolute motions themselves. If therefore, in all instances, bodies shew equal tendencies to retain their differences of relative motions, they ipso facto shew equal tendencies to retain their differennces of absolute motions, & to retain their absolute motions themselves.

How is y tendency to be observed? by putting two bodies in such a state y they cannot on account of their impenetrability, or y mutual actions of any powers both preserve their present state, or present differences of absolute motions.

...nge produced in y relative motion of y one is the effect, & y only measure of y tendency of the other to retain its present state. This change must be measured by the quantity of motion; for y perseverance is considered as a property or not of every particle of Mr in each body. The proportion therefore of these changes will indecate y proportion of y tendencies in question.

Let y following experiment be made at mid-summer noon. let A moving apparently Westward come up to B apparently at rest w y velocity of three feet in a second.

Suppose it really happens in y experement, y A meeting B at rest carries it forward w undiminished velocity. It proves y B resists no augmentation of motion but y A resists a deminution of motion, B therefore will shew no tendency to remain at rest, but A will shew a tendency to continue in motion.

On y other hand suppose that when A has come up to B it stops. That is to say B receives no encrement of motion from A, but A loses all its excess of motion. Here then B shews a tendancy to remain at rest, but A no tendency to continue in motion.

Again suppose y, after meating, both move on w y velocity of one foot in a second. Then B has received an increment of motion w may be called 1, & y is y effect & measure of A's tendency to

continue in motion. A has suffered a diminution
wh may be called 2, wh is ye effect & ye measure of Bs tendency
remain at rest. The genuine conclusion from this experi-
ment would be yt ye tendency of B to remain at rest was double
ye tendency of A to continue in motion.

Again suppose yt both move on wth the velocity of 1½ feet in a second. Then ye change in B is equal to yt in A. & therefore ye tendency of B to continue at rest is equal to the tendency of A to continue in motion.

Lastly suppose yt A=2B, & yt both move on wth ye velocity of 2 feet in a second. Then B has received an increment of 2 seconds. This is the effect & measure of ye tendency of the whole mr in A to continue in motion & half of it is ye effect & measure of ye tendency of half ye mre in A to continue in motion. A has suffered a diminution of motion wh may be called 2, & for each half has suffered a diminution 1, wh is ye effect & measure of ye tendency of ye mr in B to continue at rest. But 1 was also ye measure of ye tendency of ½A to continue in motion. therefore ½A shews ye same tendency to continue in motion yt B does to remain at rest. The just conclusion therefore from such results will be that mr has an equal tendency to continue at rest & in motion.

I chuse mid-summer noon & a westerly motion of A, yt ye relative motions might be ye simple difference of absolute motions.