## Biological Modelling BIOL 4.063/5063

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## BIOL 4063/5063 - Biological Modelling

- Lectures Tues \& Thurs, LSC 200
- No required text; readings will be available on reserve and lecture notes will sometimes be online.
- Grading : Projects, 60\% midterm exam, 20\%
final exam, 20\%

Modelling is as central to biology as it is to physics
Medelian genetics was believed to be incompatible with natural selection until dynamic models were used to explain the behaviour of genes, and statistical methods were developed to interpret the data.


## Each student should come and talk with one

 of us.
## Projects

- Each student will develop a modelling project, write it up, and present it to the class.
- This may be used in your thesis.
- Graduate students should aim for publication (this has happened with several of my students).

Example of a class project that we are after.
This one is in press in a journal (Fisheries Oceanography).



## Labs:

- We will use R (a free statistical language that is widely used).
- BUGS (also free, can be used to fit complex models).


## Historical Distinctions in Ecological Modeling

Statistical Models

$$
Y_{i j}=\mu+\tau_{j}+\beta_{i}+\varepsilon_{i j}
$$

- Emphasized linear models
- Focus on hypothesis testing
- Strong ties to data
- Weak links to biology


## Theoretical Models

$$
R_{i}^{*}=\frac{k_{i} m_{i}}{r_{i}-m_{i}}
$$

- Emphasized non-linear models
- Focus on model analysis and validation
- Weak ties to data
- Strong links to biology

The purpose is to join the two.

## Four ways to write up a model.

- English - Likelihood is the probability of observing the data given a parameter.

$$
\begin{aligned}
& \mathscr{L}\left(\theta \mid x_{1}, x_{2}, x_{3} \ldots x_{n}\right)=\prod_{i=1}^{n} c g\left(x_{i} \mid \theta\right) \\
& \mathscr{L}\left(p_{f} \mid \text { data }\right)=\prod_{i=1}^{4} p_{f}^{x_{i}}\left(1-p_{f}\right)^{n_{i}-x_{i}}
\end{aligned}
$$

- Analysis (Equations)
- Figures

Likelihood Function


- Simulations
- $\quad \operatorname{sum}(d b i n o m(46: 54,100,0.5))$
plot (k, dbinom(k, n, pi/10, log=TRUE), type='l', ylab="log density", main = "dbinom(*, $\log =$ TRUE $)$ is better than log(dbinom(*))")


## Lecture Next Thursday: I will begin to explain what likelihood methods are.

## Statistics Covered:

- Graphical Methods
- Likelihood and Bayesian Methods
- Generalized linear models
- Mixed effects models
- Meta-analysis
- Fitting dynamical models to data
- State space models


## Difference between physics and biology

- Physics has laws
- Biology has lawyers looking for loopholes.


## Why is it important to learn mathematical modelling?







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The Space Shuttle Challenger exploded shortly after take-off in January 1986. Subsequent investigation determined that the cause was failure of the O-ring seals used to isolate the fuel supply from burning gases.
NASA staff had analysed the data on the relation between ambient temperature and number of O-ring failures (out of 6), but they had excluded observations where no O-rings failed, believing that they were uninformative. Unfortunately, those observations had occurred when the launch temperature was relatively warm (65-80 degF).


There's not much data at low temperatures (the confidence band is quite wide), but the predicted probability of failure is uncomfortably high. Would you take a ride on Challenger when the weather is cold?

NASA Space Shuttle O-Ring Failures


## Look at the data

- Download the package DAAG
- Print out "orings"

| Temperature Erosion Blowby T |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 53 | 3 | 2 | 5 |
| 2 | 57 | 1 | 0 | 1 |
| 3 | 58 | 1 | 0 | 1 |
| 4 | 63 | 1 | 0 | 1 |
| 5 | 66 | 0 | 0 | 0 |
| 6 | 67 | 0 | 0 | 0 |
| 7 | 67 | 0 | 0 | 0 |
| 8 | 67 | 0 | 0 | 0 |
| 9 | 68 | 0 | 0 | 0 |
| 10 | 69 | 0 | 0 | 0 |
| 11 | 70 | 1 | 0 | 1 |
| 12 | 70 | 0 | 0 | 0 |
| 13 | 70 | 1 | 0 | 1 |
| 14 | 70 | 0 | 0 | 0 |
| 15 | 72 | 0 | 0 | 0 |
| 16 | 73 | 0 | 0 | 0 |
| 17 | 75 | 0 | 0 | 0 |
| 18 | 75 | 0 | 2 | 1 |
| 19 | 76 | 0 | 0 | 0 |
| 20 | 76 | 0 | 0 | 0 |
| 21 | 78 | 0 | 0 | 0 |
| 22 | 79 | 0 | 0 | 0 |
| 23 | 81 | 0 | 0 | 0 |

## Program

- oldpar <- par(mfrow=c(1,2))
- $\operatorname{plot}($ Total $\sim$ Temperature, data $=$ orings[c(1,2,4,11,13,18),])
- \# the observations included in the pre-launch charts
plot(Total~Temperature, data = orings)
- par(oldpar)


## Lessons for biologists:

- Don’t ignore zeros! Many students who go into biology appear to be genetically predisposed to ignore zeros.
- Distinguish between zeros, missing values, and censored values.
- The right statistical model, in this case a generalized linear model with a logit link and binomial error or a log link with a negative binomial error.
- Data + Theory
- Modelling (i.e. Information)
- Estimation (i.e Evidence)
- Action
- Like good writing, good graphical displays of data communicate ideas with clarity, precision, and efficiency.
- Like poor writing, bad graphical displays distort or obscure the data, make it harder to understand or compare, or otherwise thwart the communicative effect which the graph should convey.
- show the data
- induce the viewer to think about the substance rather than about methodology, graphic design, the technology of graphic production, or something else
- avoid distorting what the data have to say
- present many numbers in a small space
- make large data sets coherent
- encourage the eye to compare different pieces of data
- reveal the data at several levels of detail, from a broad overview to the fine structure
- serve a reasonably clear purpose: description, exploration, tabulation, or decoration
- be closely integrated with the statistical and verbal descriptions of a data set

When ever thinking about a new problem, it is worth thinking very hard about an optimal presentation of the data:


## Leatherbacks change their behaviour as the migrate south

## Proportion of time at surface Night <br> Day




## Determining the Effects of Exploitation on Shark Populations Using Fishery Dependent Data

 Julia BaumDan Kehler
Ransom A. Myers

## 13 D DALHOUSIE University



## Life history of sharks...

- slow growth rates
- late age at maturity
- low fecundity


## = Vulnerable to Overexploitation



## Life history of sharks...



Fecundity


Hammerhead spp.
Caribbean






Florida East Coast







1 Caribbean
2 Gulf of Mexico
3 Florida
4 S Atlantic Bight
5 Mid Atlantic Bight


## Data Analysis

- Assume catch follows negative binomial distribution
- Analyse positives only $\rightarrow$ zero-truncated distribution

$$
\frac{f\left(y_{T}\right)=\frac{\Gamma(y+\theta)^{y_{T}}}{\Gamma(y)}\left(\frac{\mu}{\theta+\mu}\right)^{y_{T}}\left(\frac{\theta}{\theta+\mu}\right)^{\theta}}{1-\left(\frac{\theta}{\theta+\mu}\right)^{\theta}}
$$

## Data Analysis

Parameter estimation: Generalized linear models:

## TNB with fixed $\theta$ is a one-parameter exponential family of distributions

Base model
Main effects: area, season, light sticks, temperature and year
Interactions: area*season, area*light

## Multiple tests for robustness



## Hammerhead sharks

Catch per 10000 hooks of Hammerhead Sharks


## Hammerhead

 sharks
## Sphyrna lewini



## Hammerhead sharks

Sphyrna lewini


## Thresher sharks

## Alopias spp.



## Repeat analysis using all other independent data

## Loss of sharks in the Gulf of Mexico

300_fold decline - no one noticed


Oceanic Whitetip captures per 10,000 hooks
Baum and Myers, 2004 Ecology Letters

## With training, "experts" can ignore the most obvious of data:

1872 - Man's head and leg and dolphin in stomach
1872 - 8 Great White Sharks reported caught
1888 - Woman's body and lamb in stomach
1894 - Preserved at Zagreb Nat. Hist. Mus.
1926 - Woman's shoes, laundry in stomach
1946 - Pig of 10 kg in stomach
1950 - Encounter during eating a dead calf
1954 - Attack on boat
1975+ -No sightings.

Newspaper reports of sharks in Croatia


# Plotting the data and model output 

It is impossible to underestimate the importance of plotting data.

Florence Nightingale made huge impacts because she knew enough math to understand problems, and she was a genus in the art of plotting data.

Florence Nightingale made huge impacts because she could plot data

- In 1840, Florence Nightingale begged her parents "to let her study mathematics instead of doing worsted work and practicing quadrilles."


## Florence at data t stupidity



Gauses of Mortality in the Army in the East April, 1854 to March 1855

Non-Battic
Battlo


From: F. Nightimgale, "Notes on Matters Attecting the Health, Etticiөпcy and Hospital Administration ot the British Army", 1858

## The most influential olot in edidemiologv:

5


## The 1854 London Cholera Epidemic.

- One of the first uses of a map to display epidemiological data was this dot chart (from Tufte, 1983, p. 24) by Dr. John Snow (1855) showing deaths from cholera (dots) in relation to the locations of public water pumps. Tufte says, "Snow observed that cholera occurred almost entirely among those who lived near (and drank from) the Broad Street water pump. He had the handle of the contaminated pump removed, ending the neighborhood epidemic which had taken more than 500 lives."





## Next step of plotting data: look at residuals

- Dr. Snow looked at other cases of cholera away from the Broad Street pump, and found that they either visited the area regularly, or worked in the area.
- Exceptions, e.g. residuals, made the results stronger.


Abraham Wald 1902-1950

Father of

During WWII and later in Korea and Vietnam, the U.S. Navy and Air Force studied bullet-hole patterns on returning aircraft to determine where to reinforce the aircraft against ground fire. Abraham Wald (a statistician at U.S. Center for Naval Analyses) worked on this problem from 1941. Wald dryly noted better information would have been obtained from the planes that hadn't returned. He nevertheless managed to construct statistical models which gave a useful insight into the vulnerability of different parts of the aircraft.
Decision Theory \& Sequential Analysis


Figure 6. A schematic representation of Abraham Wald's ingenious scheme to investigate where to armor aircraft.

Table 2: Admissions to Berkeley graduate programs

|  | Admitted | Rejected | Total |
| :--- | ---: | ---: | :---: |
| Males | 1198 | 1493 | 2691 |
| Females | 557 | 1278 | 1855 |
| Total | 1755 | 2771 | 4526 |

(1198/1493)/(557/1278)
$=1.84$

Sex: Male


Sex: Female


Figure 5 shows the aggregate data from Table 2. The sample ns odds ratio, Odds (Admit|Male) / (Admit|Female) is 1.84 indicating that males were almost twice as likely to be admitted. The confidence rings in the figure do not overlap, showing that this association is highly significant. Does this constitute evidence for gender bias in admission?

Sex: Male


Sex: Female
Figure 5: Fourfold display for Berkeley admissions data, margins eamated

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Table 2 is an example of a $2 \times 2$ table. For such data, the odds ratio, $\theta=n_{11} n_{22} / n_{12} n_{21}$, is a natural measure of the strength of association between the two variables.

The fourfold display depicts these frequencies by quarter circles, whose radius is proportional to $\sqrt{n_{i j}}$, so the area is proportional to the cell count (Fienberg, 1975, Friendly, 1994a,c). The cell frequencies are usually scaled to equate the marginal totals, and so that the ratio of diagonally opposite segments depicts the odds ratio. Confidence rings for the observed $\theta$ allow a visual test of the hypothesis $H_{0}: \theta=1$ corresponding to no association. They have the property that the rings for adjacent quadrants overlap iff the observed counts are consistent with the null hypothesis.

The admissions data shown in Figure 5 came from the six largest at Berkeley. To determine the source of the apparent sex bias in favor of males, we make a new plot, Figure 6, stratified by department.

Surprisingly, Figure 6 shows that, for five of the six departments, the odds of admission is approximately the same for both men and women applicants. Department A appears to differs from the others, with women approximately $2.86(=(313 / 19) /(512 / 89))$ times as likely to gain admission.

The resolution of this contradiction can be found in the large differences in admission rates among departments. Men and women apply to different departments differentially, and in these data women happen to apply in larger numbers to departments that have a low acceptance rate. The aggregate results are misleading because they falsely assume men and women are equally likely to apply in each field.


Figure 6: Fouffold display for Berkeley admissions data, by depart-

## To make these plots:

- Download the vcd package.
- data(UCBAdmissions)
- \#\# Use the Berkeley admission data.
- $\mathrm{x}<-$ aperm(UCBAdmissions, $\mathrm{c}(2,1,3)$ )
- dimnames(x)[[2]] <- c("Yes", "No")
- names(dimnames(x)) <- c("Sex", "Admit?", "Department")
- ftable(x)
- \#\# Fourfold display of data aggregated over departments, with
- \#\# frequencies standardized to equate the margins for admission
- \#\# and sex.
- `fourfold(margin.table(x, c(1, 2)))


## To make these plots:

- \#\# Fourfold display of x, with frequencies in each table
- \#\# standardized to equate the margins for admission and sex.
- fourfold(x)
- cotabplot(x, panel = cotab_fourfold)
- \#\# Fourfold display of x, with frequencies in each table
- \#\# standardized to equate the margins for admission. but not for sex.
- fourfold (x, margin = 2)

Model: (AdmitDept) (GenderDept)


Figure 7: Three-way mosaic plot for Berkeley data: Conditional independence

## Another data set: the Titanic Survival Rates

- data(Titanic)
- doubledecker(Titanic)
- doubledecker(Titanic, depvar = "Survived")
- doubledecker(Survived ~ ., data = Titanic)


Figure 20: Donner Party, fitted logistic model, $\operatorname{Pr}($ Death $) \sim$ Age + Age ${ }^{2}+$ Male

## Napoleon's Russian Campaign of 1812







Barley Yield (bushels/acre)

The figure is a Trellis display of data from an agricultural field trial of barley yields at six sites in Minnesota; ten varieties of barley were grown in each of two years. The data were presented by R. A. Fisher in The Design of Experiments and analyzed subsequently by many others.

William Cleveland's display of these data shows an apparent surprise missed by previous investigators, which occurs at the Morris site: For all other sites, 1931 produced a significantly higher overall yield than 1932. The reverse is true at Morris. But most importantly, the amount by which 1932 exceeds 1931 at Morris is similar to the amounts by which 1931 exceeds 1932 at the other sites. More displays, a statistical modeling of the data, and some background checks on the experiment led to the conclusion that the data are in error -- the years for Morris were inadvertently reversed.

The background of the data, and analysis with Trellis are described in more detail in The Visual Design and Control of Trellis Display


Barley Yield (bushels/acre)

To run this analysis:

- Load lattice package
- "barley" is a data frame with 120 observations on the following 4 variables.
- yield Yield (averaged across three blocks) in bushels/acre.
- variety Factor with levels "Svansota", "No. 462", "Manchuria", "No. 475", "Velvet", "Peatland", "Glabron", "No. 457", "Wisconsin No. 38", "Trebi".
- year Factor with levels 1932, 1931
- site Factor with 6 levels: "Grand Rapids", "Duluth", "University Farm", "Morris", "Crookston", "Waseca"


## Dotplot:

- dotplot(variety $\sim$ yield $\mid$ site, data $=$ barley, groups = year, key = simpleKey(levels(barley\$year), space = "right"), xlab = "Barley Yield (bushels/acre) ", aspect $=0.3$, layout $=c(3,2)$, ylab=NULL)
- 1


Gross Domestic Product per capita in US dollar purshasing power parity (log scale)= Money

## Tufte's Rules:

- Tone down secondary elements of a picture: layer the figure to produce a visual hierarchy.
- Replace coded labels in the figure by direct ones.
- Produce emphasis by using the smallest possible effective distinctions.
- Eliminate all unnecessary parts of a figure.
- Use small multiples: numerous repetitions of a single figure with slight variations.
- Make the graphics carry a story.

Given : depth

lat

Given : Temp


## Questions for the next few classes::

Why are fishers poor in an unregulated fishery?
When is it economically optimal to drive a whale stock to extinction?

Why is there always more pressure for quotas even when there are few fish?

Why did the cod stoc s collapse in Eastern Eanada?
Why didn't the lob ster and snow crab'stocks NOT collapse?

